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EDITORIAL

The Lorentz Transformations applied to the Problem of Two Electrons

IN the January Editorial we referred to the time-honoured question of two electrons moving side by side at high speed, which had been recently raised by a correspondent in the American journal, *Electrical Engineering*. On page 178 of this issue we publish a letter from R. E. Burgess of the National Physical Laboratory in which he refers to the principle of special relativity and applies the Lorentz transformations to the problem. We doubt very much, however, whether this resolution will satisfy the propounder of the question. As Mr. Burgess states, the ordinary physical laws are invariant, that is to say, the acceleration of the one electron away from the other is equal to the force divided by the mass after you have applied the Lorentz transformations to all the quantities concerned.

If v is the velocity of the two electrons relative to the observer, $k = (1 - v^2/c^2)^{-\frac{1}{2}}$ is slightly greater than 1.0 and the effective mass is increased in this ratio, whilst the resultant force due to the electric and magnetic fields is decreased in the same ratio. At first sight the increased mass and the decreased resultant force would suggest a decrease in the ratio of $1/k^2$ in the acceleration with which the electrons move apart, but apparently this is exactly what the acceleration would appear to be to the stationary observer. Lengths transverse to the direction of motion are unaltered but time intervals are shortened in the ratio $1/k$ and therefore the observed acceleration is modified in the ratio $1/k^2$.

In the *Philosophical Magazine* of June, 1945, there is a paper on the "Derivation of the Lorentz Transformations," by H. E. Ives of the Bell Telephone Laboratories, in which he attacks the very principle that Mr. Burgess invokes, viz., the principle of special relativity. He says, "The Lorentz transformations were obtained by Lorentz as a succession of *ad hoc* inventions to reconcile Maxwell's theory with the results of experiments on moving bodies. By Einstein they were derived after a discussion of the nature of simultaneity and the adoption of a *definition* of simultaneity which violates the intuitive and common-sense meaning of that term."

He then says: "It is proposed here to show that these transformations can be derived by imposing the laws of conservation of energy and of momentum on radiation processes as developed by Maxwell's methods. To do this we shall study the interaction of radiation and matter, specifically the impact of a train of radiation on a perfectly reflecting particle. The energy and momentum of the radiation will be obtained from electromagnetic wave theory; the resultant momentum and energy of the particle will be obtained by requiring the conservation of momentum and energy. It will be found that there is an apparent discrepancy in the equations; the resolution of this discrep-

ancy demands that the *mass* of the particle vary with velocity. Next, the same impact experiment will be considered with the system in uniform motion. Again, a discrepancy will be encountered, and the resolution of this demands that the *lengths* and *time intervals* also vary with velocity . . . From these variations of dimensions the Lorentz transformations can be derived. *The space and time concepts of Newton and Maxwell are retained without alteration.*"

He finds that the mass M when moving is equal to $M_0(1 - v^2/c^2)^{-\frac{1}{2}}$ as stated above. He then assumes that the experiment is carried out on a platform *moving with respect to the wave transmitting medium.* This is presumably our old friend the ether, but Dr. Ives gives no indication as to the method of determining whether a body is moving with respect to it or not. We are reminded of the Kelvin Lecture given in 1925 by Sir J. H. Jeans on 'Electric Forces and Quanta'; in moving a vote of thanks Sir Oliver Lodge —who was a great believer in mediums— remarked that although the lecturer had *expressed* the facts of nature in a wonderful manner without referring to a medium, he had not *explained* them.

We cannot help feeling that Dr. Ives's argument is seriously weakened by this statement without any explanation. As the result of his calculations he finds that frequencies and lengths in the direction of motion are reduced in the ratio $1/k$, in fact, he always gets the right answer, which leads one to suspect that by 'moving with respect to the medium,' he means 'moving with respect to the observer,' and that every observer has his own medium, but it is not at all clear.

In discussing his method, Dr. Ives says: "The derivation of the Lorentz transformations here given has for its central feature the confronting of the Maxwell electromagnetic picture of radiation with the laws of conservation of energy and momentum. The derivation here given is strictly according to the tenets of classical physics. . . . No hybrid 'space-time' is invoked. . . . The indeterminances and impotences by which the 'Restricted Theory of Relativity' has been widely publicized . . . all follow as consequences of our resolution of the apparent conflict between the laws of the conservation of energy and momentum on the one hand and the laws of light propagation on the other, our resolution being in favour of the conservation laws. . . . The fact that the Lorentz transformations with all their consequences are deducible from these, classes the 'Special Principle of Relativity' as a superfluous hypothesis."

It is certainly very reassuring to know that Dr. Ives arrives at exactly the same results; it makes one wonder whether he is really adopting a different line of attack or merely clothing it in different words. This suspicion is strengthened by the vagueness of his references to movement relative to the medium.

G. W. O. H.

The Permeability of Iron-Dust Cores

IN this number we publish an article by R. E. Burgess which deals with the permeability of iron-dust cores. The author says "If the mass core contains uniform spherical particles of iron of permeability μ_0 insulated from each other, the effective permeability of the core is given by

$$\mu = \frac{(\mu_0 + 2) + 2p(\mu_0 - 1)}{(\mu_0 + 2) - p(\mu_0 - 1)} = \frac{1 + 2pM_0}{1 - pM_0}$$

where p is the fractional volume of the iron and $M_0 = (\mu_0 - 1)/(\mu_0 + 2)$. This equation can also be written in the form

$$\frac{\mu - 1}{\mu + 2} = p\frac{\mu_0 - 1}{\mu_0 + 2} = pM_0$$

In *Wireless Engineer* of January 1933 we considered this question and developed a formula on the assumption that the iron particles were cubes arranged symmetrically with parallel faces. The formula obtained was

$$\mu = \frac{\mu_0 - 2\delta(\mu_0 - 1)/3}{1 + (\mu_0 - 1)\delta/3}.$$

where δ is the fraction of the space occupied by insulating material, i.e. $\delta = 1 - p$.

Out of curiosity we calculated several points on Fig. 1 of Mr. Burgess's article to see how the values calculated on the cubical assumption compared with those calculated on the spherical assumption. To our surprise the values agreed so exactly that we suspected that the two formulae were really one and the same, and so it turned out. Putting $p = 1 - \delta$ we have

$$\mu = \frac{1 + 2pM_0}{1 - pM_0} = \frac{1 + 2(1-\delta)(\mu_0 - 1)/(\mu_0 + 2)}{1 - (1-\delta)(\mu_0 - 1)/(\mu_0 + 2)}$$
$$= \frac{(\mu_0 + 2) + 2(1-\delta)(\mu_0 - 1)}{(\mu_0 + 2) - (1-\delta)(\mu_0 - 1)}$$
$$= \frac{3\mu_0 - 2\delta(\mu_0 - 1)}{3 + \delta(\mu_0 - 1)} = \frac{\mu_0 - 2\delta(\mu_0 - 1)/3}{1 + (\mu_0 - 1)\delta/3}$$

Hence whether the particles are assumed to be spherical or cubical, the approximations that are made lead to the same final formula.

Although in 1933 we thought that the cubical assumption was original and that all previous writers had assumed spherical or spheroidal particles, we subsequently discovered that Doebke, to whose work we gave a reference, had already in 1930 calculated the permeability for the cubical particles and had obtained a formula which was really the same as we obtained in 1933.

G. W. O. H.

SERIES-RESONANT CRYSTAL OSCILLATORS*

By F. Butler, B.Sc., A.M.I.E.E.

SUMMARY.—After a discussion of the equivalent circuit of a vibrating quartz crystal, associated with external series or parallel reactances, a number of new oscillator circuits are described. These all employ series resonance of the quartz crystal. Their frequency of operation is found to be slightly affected by associated external series reactance, in contrast with oscillators of the type using parallel crystal resonance, which are affected by reactances in parallel with the crystal. It is finally suggested that the same fundamental circuit principles can be employed in the design of crystal filters.

Introduction

ANALYSIS of the equivalent circuit of a vibrating quartz crystal shows that it is capable of simulating a series-tuned circuit at one particular frequency and that it exhibits parallel resonance at a slightly higher frequency. Most of the early crystal oscillators employed the parallel-resonant mode, the frequency of which is dependent on associated parallel reactances, e.g. the holder capacitance. More recently, oscillators have been developed using series resonance. These include the Meacham bridge-stabilized oscillator, and a two-valve circuit previously described by the writer. The frequency of these oscillators is scarcely affected by impedance in parallel with the crystal, but is sensitive to the effect of reactance in series with it. The attainable precision of frequency with the series-resonant oscillator is higher than that possible with the parallel-resonant alternative. The circuits to be described do not compare in frequency stability with the Meacham oscillator, but they deliver a very much larger power output.

* MS. accepted by the Editor, January 1946.

Equivalent Circuit of Quartz Crystal

Fig. 1 shows the electrical equivalent of a piezo-electric crystal in which :—

L = effective inductance of the crystal.
C = effective capacitance.
C_1 = holder capacitance.
X_s = series reactance (air-gap effects, wiring inductance or inserted reactance).

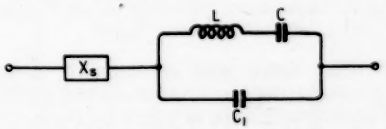

Fig. 1. *Equivalent circuit of a piezo-electric crystal.*

Neglecting resistances, analysis of this circuit shows that parallel resonance occurs at the frequency given by :—

$$\omega^2 = \frac{1}{L}\left(\frac{1}{C} + \frac{1}{C_1}\right)$$

It is clearly affected by the holder capacitance. The condition for series resonance is given by the following expression :—

$$\left(j\omega L + \frac{1}{j\omega C}\right) = \frac{-X_s/j\omega C_1}{X_s + 1/j\omega C_1}$$

In the particular case when X_s is due to an inductance L_1, $X_s = j\omega L_1$, and the condition becomes :—

$$\left(\omega L - \frac{1}{\omega C}\right) = \frac{L_1/C_1}{\omega L_1 - 1/\omega C_1}$$

The true series resonant frequency of the crystal alone is, therefore, only attained if L_1 is zero or if the inductive reactance is cancelled by tuning out with additional series capacitance.

If X_s is a capacitive reactance, given by $1/j\omega C_2$ then :—

$$\left(\omega L - \frac{1}{\omega C}\right) = \frac{1}{\omega(C_1 + C_2)}$$

The series-resonance frequency is therefore more closely approached as the external series inductance tends to zero or as the external capacitance is indefinitely increased.

Influence of Associated Circuits

In some of the oscillator circuits to be described, the crystal will be placed in series with the untuned primary of an R.F. transformer of which the secondary is tuned. It is of interest to calculate the input impedance and to estimate the probable effect on the series-resonant frequency of the crystal-transformer combination.

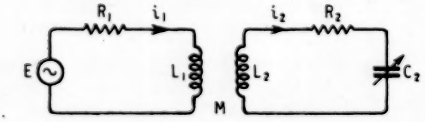

Fig. 2. *Equivalent circuit of an R.F. transformer.*

Writing down and solving the network equations, using the symbols shown in Fig. 2, the input impedance is given by :—

$$E/i_1 = Z_i = \left\{R_1 + \frac{\omega^2 M^2}{R_2^2 + (\omega L_2 - 1/\omega C_2)^2} \cdot R_2\right\} + j\left\{\omega L_1 - \frac{\omega^2 M^2}{R_2^2 + (\omega L_2 - 1/\omega C_2)^2} \cdot (\omega L_2 - 1/\omega C_2)\right\}$$

For secondary resonance, the equation simplifies to :—

$$Z_i = R_1 + \frac{\omega^2 M^2}{R_2} + j\omega L_1$$

The real component does not affect the frequency. The reactive term lowers the frequency below that of the crystal. Slight mistuning of the secondary can be used to cancel the inductive reactance $j\omega L_1$, this condition being indicated by maximum R.F. crystal current. To estimate the probable effect of the reactance $j\omega L_1$ on the final frequency, the electrical constants of the crystal must be known. For a high-frequency crystal, the effective inductance is commonly of the order of one henry. The inductance L_1 is unlikely to exceed about 10 μH, so that the frequency change can be expected to be about five parts in one million. Mistuning of the secondary puts larger transferred reactances in series with the crystal and causes proportionately greater frequency changes, particularly when the transferred reactance is inductive.

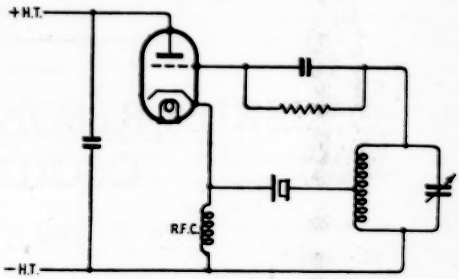

Fig. 3. *"Earthed anode" type of Hartley oscillator.*

Practical Series-Resonant Oscillator Circuits

The three basic circuits to be discussed are shown in Figs. 3, 4 and 5. All involve the use of a cathode-driven amplifier, connected regeneratively to ensure self-oscillation. They are merely variations of the Hartley circuit, with the inclusion of a quartz crystal between the cathode tap or coupling coil and the valve cathode. The input impedance of the amplifier and the output impedance of the tapped coil or R.F. transformer are both low in value. At series resonance, the crystal appears as a low resistance and permits the maintenance of oscillations. At off-resonance frequencies the crystal simulates a high reactance linking the low-impedance source and load. Attenuation and phase shift occur and oscillations cease.

The radio-frequency choke in each cathode lead serves merely to provide a D.C. path

for the valve anode current without short-circuiting the R.F. driving voltage. Its value is not critical. In low-power oscillators, a resistance of the order of 20,000 ohms may be substituted provided that the grid leak is in this case returned to the cathode so as to avoid over-biasing the valve.

As they stand, all three circuits are only suitable for use with low-frequency crystals. Subject to this limitation, they give a satisfactory performance, and oscillate readily even with poor quality crystals, difficult to operate in standard circuits. Figs. 3 and 4 are convenient for use with symmetrically-mounted crystals of the reversible plug-in type. Fig. 5 is best for use with precision crystals of which the holders are usually designed to have one side earthed.

At high frequencies it is found that the holder capacitance (C_1 in Fig. 1) exercises a shunting effect and oscillations occur at frequencies uncontrolled by the crystal. There are two standard methods of dealing

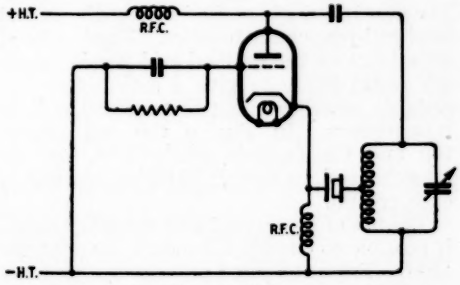

Fig. 4. *" Earthed grid " Hartley oscillator.*

with difficulties of this kind. Neutralization may be employed, or alternatively, the undesired capacitance can be tuned out by parallel inductance, in which case the capacitive reactance is removed, and a high resistance substituted. In the present case, the neutralization technique will be employed.

Figs. 6 and 7 show practical circuits. In the first a neutralizing winding is coupled to the main tuned circuit, and is preferably wound on the same coil former. The number of turns should be equal to those included between the crystal tap and earth. In this case, stability is ensured at all frequencies, other than at the desired series resonance frequency of the crystal, when the neutralizing capacitor setting is equal to the crystal holder capacitance. Over-neutralization gives a measure of control of the R.F. oscillator excitation voltage. Fig. 7 shows a circuit which has the practical advantage that one side of both crystal and neutralizing capacitor can be earthed. In both cases, the neutralizing-bridge balance is upset at the series-resonance frequency of the crystal. At this frequency the crystal resistance is very low and regenerative feedback occurs,

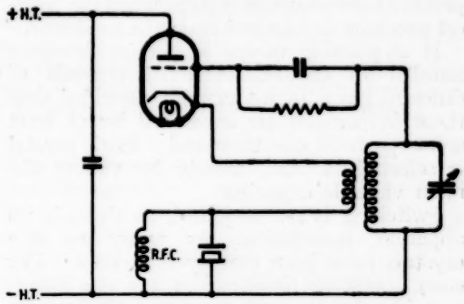

Fig. 5. *" Earthed anode " oscillator with cathode-circuit reaction coil.*

sufficient to start oscillations. In Fig. 6 the crystal-tapping point on the main inductance should be positioned to include one-fifth to one-tenth of the total turns on the main winding. The corresponding coupling coil in Fig. 7 should be similarly proportioned. In this circuit, an earthed electrostatic screen may be provided between the two windings.

A feature of all the oscillator circuits so far described is that the correct tuning point is indicated, with leak and condenser bias, when the valve anode current falls to a

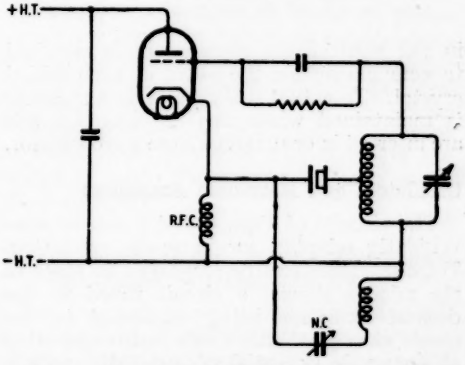

Fig. 6. *A neutralized Hartley oscillator of the " earthed anode " type.*

minimum. This is in contrast with conventional crystal circuits, which become unstable as this condition is approached. The dip

in anode current is pronounced and sharply defined, though the narrow crevasse is not quite symmetrical. Measurements show that minimum anode current does not exactly correspond to maximum radio-frequency current through the crystal. The latter is the correct setting, but the difference is of no practical consequence, except when the highest precision is required from the oscillator.

It is possible to use in the oscillators a number of parallel-connected crystals of different resonant frequencies, provided that these frequencies are separated by at least a few parts in one thousand. Each crystal is selected in turn, merely by tuning the main variable capacitor.

Switching is not required, as there is no apparent interaction. As many as five crystals have been employed at once. For each operating frequency, there is a corresponding dip in anode current. An increase

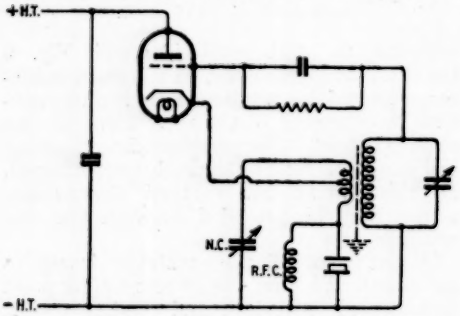

Fig. 7. *With a cathode-circuit reaction coil neutralization of the crystal holder capacitance can be effected in the manner shown here.*

in the neutralizing capacitance is required to compensate for the effect of each added crystal. In actual practice, perfect stability is maintained when two or three crystals are inserted in or removed from the oscillator.

Oscillators and Harmonic Amplifiers

The circuits of Figs. 6 and 7 can be conveniently adapted for harmonic operation. Tetrode valves can be employed in place of the triodes shown, a circuit tuned to the desired harmonic being connected in the anode circuit. With a 6F6 valve operating at an anode potential of 400 volts, with a 200-volt screen supply, a grid leak of 30,000 ohms and a grid capacitor of 100 micro-micro-farads the radio-frequency crystal current is 80 mA. These figures were obtained with a high-grade crystal of 2,600 kc/s, the third harmonic being selected in the anode circuit. Under these conditions, the D.C. anode current was 25 mA.

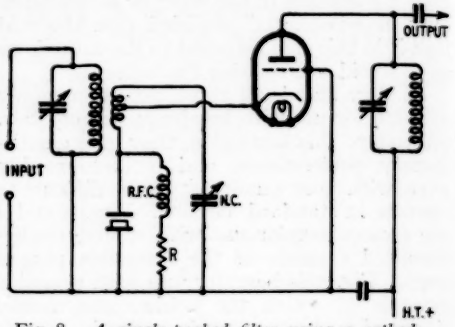

Fig. 8. *A single-peaked filter using a cathode-input stage can take this form.*

Crystal-Filter Circuits

No experimental work has been done on quartz filters but two circuits are suggested in Figs. 8 and 9 which show some promise. The first is a single-peaked filter ; the second has band-pass characteristics. The low input impedance of the earthed-grid triode amplifier seems likely to give a satisfactory impedance match without the use of auxiliary transformers. In Fig. 9 the coil centre-tap and the cathode connections may be interchanged to permit earthing one side of each crystal.

The use of an earthed-grid amplifier makes it possible to employ a triode valve with the filter. From the point of view of impedance matching and amplifier gain, a pentode may

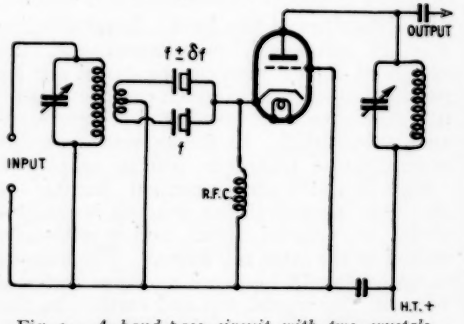

Fig. 9. *A band-pass circuit with two crystals does not require neutralizing if the shunt capacitances on the two crystals are identical.*

be preferred. Cathode bias may be used, the correct value being obtained, if necessary, by the inclusion of a resistance R to supplement that of the R.F. choke.

EQUIVALENT CAPACITANCES OF TRANSFORMER WINDINGS*

By W. T. Duerdoth, B.Sc., A.M.I.E.E.

(*Post Office Research Station*)

SUMMARY. The paper shows that the distributed capacitances between windings, or windings and screens, of transformers may be represented by lumped capacitances provided that the magnetic coupling between the turns of a winding is perfect. Expressions have been obtained for the equivalent capacitances of a number of different arrangements including windings in layers, sections and with screens.

1. General

A KNOWLEDGE of the equivalent capacitance of a transformer winding is essential if the performance of the transformer is to be predictable. This knowledge is particularly necessary when the transformer has to pass a wide band of frequencies, as often occurs in telecommunication work. It is sometimes desirable to design the transformer as a filter section giving low-pass or band-pass characteristics as may be required, in which case the leakage inductance, shunt inductance and self-capacitances of the windings can be employed together with added components to form the elements of the filter. By these means, a maximum band-width can be passed without frequency distortion. This paper, however, considers only the equivalent capacitances of the winding.

Capacitance between transformer windings, and between windings and screens, consists of distributed capacitances along the whole windings and the effect of these capacitances depends upon the voltages between the corresponding points on the windings or screens.

It is possible to express the distributed capacitance as a lumped capacitance across the whole winding and relate this lumped capacitance to a total capacitance which can be easily measured at a low frequency where the inductance of the winding can be neglected.

Capacitances between turns on the same layer of a winding also exist but these are usually negligible in comparison with inter-layer and layer to screen capacitances.

If it is assumed that the windings or screens form coaxial cylinders, then it becomes necessary to estimate the capacitance C between such cylinders and in this analysis the following expressions have been employed.

$$C = \frac{0.0885kSD}{\Delta}\ \mu\mu\text{F} \quad .. \quad .. \quad (1)$$

Where k = permittivity of the dielectric
S = mean circumference (cm)
D = length of cylinder (cm)
Δ = distance between cylinders (cm)

This is true when the dimensions are such that the capacitor may be regarded as parallel plates extending to infinity but will be in error in cases where D or S are not large compared with Δ in which case the following will be more accurate.

$$C = \frac{1.1kD}{2\log_e r_2/r_1}\ \mu\mu\text{F} \quad .. \quad .. \quad (2)$$

This is true for two coaxial cylinders of radii r_1 and r_2 extending to infinity.

Both (1) and (2) are in error when finite capacitors are considered, since a modified field occurs at the boundaries of the plates and gives rise to an increased capacitance; this effect is usually referred to as "fringing," but will not be of importance when DS is large compared with Δ.

However, this limitation must be borne in mind when practical measurements are compared with the theoretical expressions obtained below.

2. Equivalent Circuit for an Unbalanced Single-layer Winding with Screen

Fig. 1 represents a single-layer winding AB with one end B connected to an adjacent screen. When connection B is broken, the total winding-to-screen capacitance C_0 can be measured at a low frequency. Alternatively, C_0 may be calculated using expressions (1) or (2).

An equivalent circuit can be developed

* MS accepted by the Editor, January 1946.

representing the transformer. The following symbols will be employed :

L = total length of wire in winding
x = length of wire between section δx and B
l = length of one turn
N = number of turns
R = a constant, proportional to the reluctance of the magnetic circuit.
L_0 = inductance $= N^2/R$.

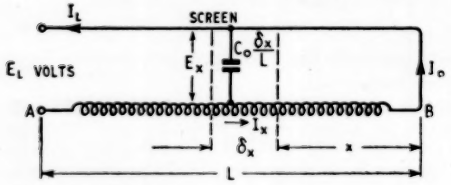

Fig. 1 (above). *A single-layer winding AB has the end B connected to a screen.*

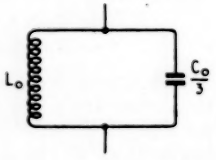

Fig. 2 (right). *In the equivalent circuit the effective capacitance is one-third of the total.*

Assume that all turns are perfectly coupled, i.e., the same flux is linked with all turns or the field strength is uniform throughout the coil. Then the voltage falls linearly along the winding and

$$E_x = E_L\, x/L \qquad .. \qquad .. \qquad .. \qquad (3)$$

Then flux produced by a small length of wire $\delta x = \dfrac{I_x \delta x}{Rl}\, 10^8$ maxwells.

Therefore, total flux

$$= \frac{1}{Rl}\int_0^L I_x dx \times 10^8 \text{ maxwells} \qquad .. \quad (4)$$

Thus for sinusoidal excitation

$$\text{Applied e.m.f.} = E_L = j\omega \frac{N}{Rl}\int_0^L I_x\, dx \qquad (5)$$

Considering a small section

$$\frac{dI_x}{dx} = E_x \frac{j\omega C_0}{L} \qquad .. \qquad .. \qquad .. \quad (6)$$

Then from (3), (5) and (6)

$$\frac{dI_x}{dx} = \frac{j\omega C_0 x E_L}{L^2}$$

$$\therefore \quad I_x = \frac{j\omega C_0 x^2 E_L}{2L^2} + K \qquad .. \qquad .. \quad (7)$$

where K is a constant of integration.

Substitute (7) in (5)

$$E_L = \frac{j\omega N}{Rl}\int_0^L \left(j\omega \frac{C_0 E_L x^2}{2L^2} + K\right) dx$$

$$= \frac{j\omega N}{Rl}\left[\frac{j\omega C_0 x^3}{6L^2} E_L + Kx\right]_0^L$$

$$= \frac{j\omega L_0}{Nl}\left(\frac{j\omega C_0 L}{6} E_L + KL\right)$$

$$= jwL_0\left(\frac{j\omega C_0}{6} E_L + K\right)$$

$$\therefore \quad K = -E_L\left(\frac{j\omega C_0}{6} - \frac{1}{j\omega L_0}\right)$$

$$\therefore \quad I_x = \frac{j\omega C_0 x^2}{2L^2} E_L - E_L\left(\frac{j\omega C_0}{6} - \frac{1}{j\omega L_0}\right)$$

$$= \left(\frac{j\omega C_0}{3} + \frac{1}{j\omega L_0}\right) E_L$$

$$Z_L = \frac{E_L}{I_L} = \frac{j\omega L_0}{1 - \omega^2 L_0 C_0/3} \qquad .. \quad (8)$$

The equivalent circuit is shown in Fig. 2 and the effective capacitance is $\frac{1}{3}$ of the total capacitance.

We also have

$$\frac{I_0}{E_L} = -j\omega \frac{C_0}{6} + \frac{1}{j\omega L_0} \qquad .. \qquad .. \quad (9)$$

Where I_0 is the current at B.

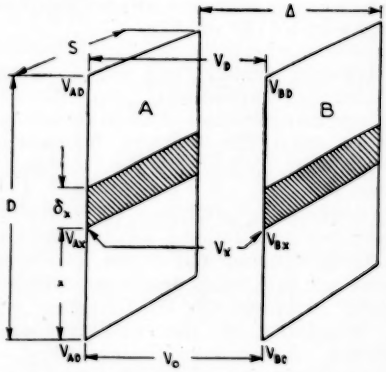

Fig. 3. *Adjacent screens or windings can be represented by plates A and B.*

We can thus regard the winding as a parallel capacitance and inductance, provided that the coupling between each turn of the winding is perfect. However, at a very high frequency the leakage flux and the capacitance between adjacent turns become

of importance, and the impedance of the winding passes through a number of series and parallel resonances.

In the above treatment, we have proved that the distributed capacitance is equivalent to a lumped capacitance across the whole winding. Only the simple case of a winding with one end connected to the screen has been treated, but capacitances between windings at different alternating potentials could be treated in a similar way. However, if we assume that all cases are reducible to a simple lumped capacitance, then the following known method based on energy considerations only is more suitable for determining its magnitude.

3. Calculation of Equivalent Lumped Capacitance

Consider two plates A and B representing two adjacent screens or windings of area $D \times S$, as shown in Fig. 3.

$$\text{Total capacitance} = 0.0885\frac{kSD}{\Delta} = C_0\ \mu\mu\text{F} \quad \ldots \quad (10)$$

$$\text{Energy stored} = W = \frac{C_0V^2}{2} \text{ joules,}$$

when the potential difference is V over the whole area.

Consider the case where the potential difference V_x is a linear function of x.

Then the energy stored in δx is given by

$$\delta W = \frac{0.0885kS}{\Delta} \cdot \frac{\delta x}{2} \cdot V_x^2 \times 10^{-12} \text{ joules}$$

Total energy

$$= W = \frac{0.0885kS}{2\Delta} \int_0^D V_x^2\,dx \times 10^{-12} \text{ joules}$$

$$V_x = V_{Bx} - V_{Ax} = \left[V_{B0} + (V_{BD} - V_{B0})\,x/D\right] - \left[V_{A0} + (V_{AD} - V_{A0})\,x/D\right] \quad \ldots \quad (11)$$

(This assumes that the voltage changes linearly as assumed to be the case in the last section.)

$$\therefore \quad V_x = V_0 + (V_D - V_0)\,x/D$$

$$\int_0^D V_x^2\,dx = \int_0^D [V_0 + (V_D - V_0)\,x/D]^2\,dx$$

$$= (D/3)\left[V_0^2 + V_0V_D + V_D^2\right]$$

$$\therefore \quad W = \frac{0.0885kS}{2\Delta} \cdot (D/3)\left[V_0^2 + V_0V_D + V_D^2\right] 10^{-12} \text{ joules}$$

If one of the windings has an applied voltage of V_p then the equivalent lumped capacitance is given by

$$C_p = \frac{2W}{V_p^2} 10^{12}\ \mu\mu\text{F} = \frac{0.0885kS}{\Delta V_p^2} \cdot D/3\left[V_0^2 + V_0V_D + V_D^2\right]\mu\mu\text{F}$$

$$C_p = \frac{C_0}{3V_p^2}(V_0^2 + V_0V_D + V_D^2) \quad \ldots \quad (12)$$

When $V_0 = 0$ and $V_D = V_p$ as in the previous section, then $C_p = C_0/3$.

4. Equivalent of Interlayer Capacitance of Single-layer Transformers

In this section the following additional symbols are employed :—

V_p = voltage on primary winding
V_s = voltage on secondary winding
$N = V_s/V_p$
C_p = equivalent lumped capacitance across primary
C_s = equivalent lumped capacitance across secondary
C_0 is the low-frequency capacitance measured between the primary winding and screen or secondary windings with ends disconnected.

Case A. Unbalanced windings without screen, earth connections at similar ends ; Fig. 4(*a*).

$$\text{From equation (12)} \quad C_p = \frac{C_0}{3}\frac{(+V_s - V_p)^2}{V_p}$$

$$C_p = (N-1)^2\,C_0/3$$

If secondary is wound in same direction as primary, as shown, then sign of V_s is positive and vice versa.

If N is small then $C_p \approx C_0/3$
If N is large then $C_p \approx N^2C_0/3$
or $C_s \approx C_0/3$

If N is unity then $C_p = 0$ if the secondary is wound in the same direction as the primary, which is self evident since only capacitance between layers is considered.

Case B. Unbalanced windings without screen, earth connected to opposite ends ; Fig. 4(*b*).

$$C_p = [V_0^2 + V_0V_D + V_D^2]\,C_0/(3V_p^2)$$

$$C_p = [1 + N + N^2]\,C_0/3$$

If the secondary is wound in the same direction as the primary, then the sign of V_s is negative, as shown.

If N is small then $C_p \approx C_0/3$

If N is large then $C_p \approx N^2 C_0/3$

or $C_s = C_0/3$

If N is unity then $C_p = C_0$

if the windings are in the same direction.

Case C. Unbalanced windings with screen, earths at similar ends ; Fig. 4(*c*).

Assuming the screen to be equidistant from both windings.

$$C_p = [1 + N^2]\, C_0/3$$

If N is small then $C_p = C_0/3$

If N is large then $C_p \approx N^2 C_0/3$

or $C_s = C_0/3$

If N is unity then $C_p = 2C_0/3$

Case D. Unbalanced windings with screen, earth to opposite ends of windings ; Fig. 4(*d*) Identical to *Case C.*

Case E. Unbalanced to balanced windings without screen ; Fig. 4(*e*).

$$C_p = [1 - N/2 + N^2/4]\, C_0/3$$

When N is small $C_p = C_0/3$

When N is large $C_s = C_0/12$

or $C_p = C_0 N^2/12$

When N is unity $C_p = C_0/4$

Advantage can be obtained by winding both windings in same direction.

Case F. Balanced to unbalanced windings with screen ; Fig. 4(*f*).

$$C_p = (1 + N^2/4)\, C_0/3$$

Case G. Balanced to balanced windings unscreened ; Fig. 4(*g*).

$$C_p = (1 - N)^2 C_0/12$$

With appropriate choice of sign $C_p = 0$ when $N = 1$.

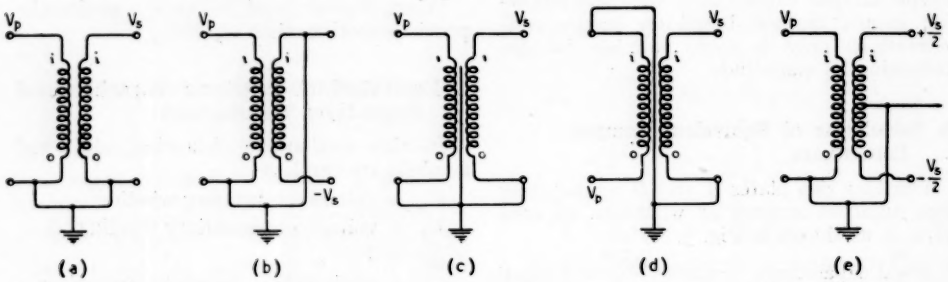

Fig. 4. *Various arrangements of winding connections and screens are shown : (a) Windings unscreened and unbalanced ; similar ends earthed ; (b) windings unscreened and unbalanced, opposite ends earthed ; (c) windings screened and unbalanced ; similar ends earthed; (d) windings screened and unbalanced; opposite ends earthed ; (e) windings unbalanced to balanced ; no screen ; (f) windings unbalanced to balanced ; with screen ; (g) windings balanced to balanced ; no screen.*

5. Multilayer Coils

5.1 *Multilayer Coil without a Screen.*

Consider a coil, as in Fig. 5, consisting of n layers each separated by Δ_1. Area of each layer $= SD$. Let $C_0 = L.F.$ capacitance between any two adjacent layers when the interconnection is broken.

The total equivalent lumped capacitance may be estimated by summing the energies stored between each pair of layers.

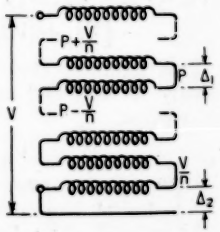

Fig. 5. *Capacitance of a multilayer coil.*

$$W = \frac{2}{3n^2} \cdot \frac{KV^2}{\Delta_1}$$

Where $K = 0.0885SDk$ (The value V_D of Fig. 3 is $2V/n$ in this case.)

There are $(n-1)$ pairs of layers

$$\therefore \quad \Sigma W = \frac{2(n-1)}{3n^2} \cdot \frac{KV^2}{\Delta_1}$$

and since $C_0 = K/\Delta_1$

The equivalent lumped capacitance C is given by

$$C = \frac{2\Sigma W}{V^2}$$

$$= \frac{4C_0}{3n^2}(n-1) \quad \ldots\ldots\ldots \text{ from (12)}$$

5.2 *Multilayer Coil with Screen connected to the Low-potential end.*

Consider the case where a screen spaced Δ_2 from the first layer is connected to the low potential end of the winding.

Energy stored between first layer and screen

$$W_1 = \frac{1}{6n^2} \cdot \frac{KV^2}{\Delta_2}$$

Energy stored between the layers

$$\Sigma W_2 = \frac{2(n-1)}{3n^2} \cdot \frac{KV^2}{\Delta_1}$$

$$\therefore \quad C = C_0\left[\frac{4(n-1)}{3n^2} + \frac{1}{3n^2} \cdot \frac{\Delta_1}{\Delta_2}\right]$$

If $\Delta_1 = \Delta_2$

$$C = \frac{4n-3}{3n^2} C_0$$

6. Multilayer Coils with Sections

6.1 *Without Screen*

Where there are G sections, each of n layers, but no screen, then by a similar method we have :—

Energy stored between pair of layers

$$= \frac{2KV^2}{3(Gn)^2\Delta_1}$$

Total energy for each section

$$= \frac{2KV^2(n-1)}{3(Gn)^2\Delta_1}$$

Total energy for G sections

$$= \frac{2KV^2(n-1)G}{3(Gn)^2\Delta_1}$$

$$\therefore \quad C = \frac{4(n-1)}{3Gn^2} C_0$$

6.2. *With a Screen*

A general expression for a coil wound on to a screen and divided into sections each containing several layers is very involved, but the particular case shown in Fig. 6 of four sections with n layers each, is treated below :—

Energy stored between screen and

$$\text{section A} = \frac{KV^2}{6\Delta_2} \cdot \frac{1}{16n^2}$$

Energy stored between screen and

$$\text{section B} = \frac{KV^2}{6\Delta_2} \cdot \frac{3n^2+3n+1}{16n^2}$$

Energy stored between screen and

$$\text{section C} = \frac{KV^2}{6\Delta_2} \cdot \frac{12n^2+6n+1}{16n^2}$$

Energy stored between screen and

$$\text{section D} = \frac{KV^2}{6\Delta_2} \cdot \frac{27n^2+9n+1}{16n^2}$$

Total energy stored between screen

$$\text{and coil} = \frac{KV^2}{6\Delta_2} \cdot \frac{3(14n^2+6n+1)}{16n^2}$$

Total energy stored between layers

$$\text{of coil} = \frac{KV^2}{6\Delta_1} \cdot \frac{n-1}{n^2}$$

$$\text{Total energy} = \frac{KV^2}{6}\left[\frac{n-1}{\Delta_1 n^2} + \frac{3(14n^2+6n+1)}{\Delta_2 16n^2}\right]$$

$$\therefore \quad C = \frac{C_0}{3}\left[\frac{n-1}{n^2} + \frac{3(14n^2+6n+1)\Delta_1}{16n^2\Delta_2}\right]$$

If $\Delta_1 = \Delta_2$

$$C = C_0 \frac{42n^2+34n-13}{48n^2}$$

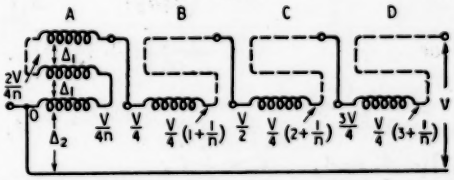

Fig. 6. *Capacitance of a coil of 4 sections with a screen.*

7. Disposition of Balanced Winding with respect to a Screen

In order to compare the various methods of winding balanced coils, C_0 has been chosen as the capacitance to a single screen of the whole winding laid out a distance Δ_2 from the screen. Thus, if we assume a

constant screen area, the value of C_0 for a winding of n layers will be n times that of a single layer winding.

7.1 *Single Screen Only.*

Case A. Fig. 7(*a*).

This case has been dealt with in section 3.

$$C = C_0/12$$

Case B. Fig. 7(*b*).

Let Δ_2 be spacing between adjacent windings

Then energy, coil to screen $= \dfrac{KV^2}{48\Delta_2}$

Energy between layers in coil $= \dfrac{KV^2}{12\Delta_1}$

Total energy $= \dfrac{KV^2}{12}\left[\dfrac{1}{4\Delta_2} + \dfrac{1}{\Delta_1}\right]$

$$\therefore \quad C = \frac{K}{6\Delta_2}\left[\frac{1}{4} + \frac{\Delta_2}{\Delta_1}\right]$$

or $$C = \frac{C_0}{24}\left[\frac{1}{4} + \frac{\Delta_2}{\Delta_1}\right] \ldots .$$

Since $C_0 = \dfrac{4K}{\Delta_2}$

If $\Delta_1 = \Delta_2$ $\quad C = \dfrac{5}{96}C_0$

Case C. Fig. 7(*c*).

Energy, coil to screen $= \dfrac{KV^2}{48\Delta_2}$

Energy, between layers $= \dfrac{13KV^2}{48\Delta_1}$

Total energy $= \dfrac{KV^2}{48}\left[\dfrac{1}{\Delta_2} + \dfrac{13}{\Delta_1}\right]$

$$\therefore \quad C = \frac{K}{24\Delta_2}\left[1 + 13\frac{\Delta_2}{\Delta_1}\right]$$

or $$C = \frac{C_0}{96}\left[1 + 13\frac{\Delta_2}{\Delta_1}\right] \ldots$$

Since $C_0 = \dfrac{4K}{\Delta_2}$

If $\Delta_1 = \Delta_2$ $\quad C = \dfrac{7}{48}C_0$

7.2 *Screen on Both Sides of Winding.*

In this section, a fixed amount of winding space is assumed, i.e. the space between two screens distance Δ apart and having a capacitance $C_1 = \dfrac{0.0885kSD}{\Delta}$ $\mu\mu$F.

Case A. Fig. 8(*a*).

Single-layer winding $C = C_1/3$.

Case B. Fig. 8(*b*)

Two layer winding.

Energy, coil to screen $= \dfrac{KV^2}{12\Delta_2}$

Energy between layers $= \dfrac{KV^2}{6\Delta_1}$

Total energy $= \dfrac{KV^2}{6}\left[\dfrac{1}{2\Delta_2} + \dfrac{1}{\Delta_1}\right]$

$$\therefore \quad C = \frac{K}{3}\left[\frac{1}{2\Delta_2} + \frac{1}{\Delta_1}\right]$$

or $$C = \frac{1}{3}\left[\frac{1}{2\Delta_2} + \frac{1}{\Delta_1}\right]\Delta C_1$$

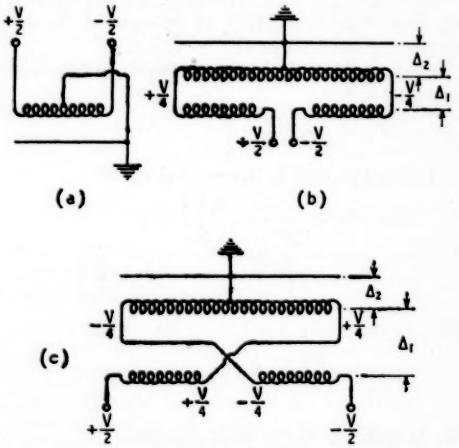

Fig. 7. *The capacitance of single-screened balanced windings: (a) plain winding; (b) sectionalized winding; (c) cross-connected sectionalized winding.*

If Δ_1 and Δ_2 are varied with Δ constant, then a minimum capacitance occurs when $\Delta_1 = \Delta/2$ and $\Delta_2 = \Delta/4$

When $C_{\text{min.}} = 4C_1/3$

Case C. Fig. 8(*c*).

Two-layer winding

Energy between layers $= \dfrac{KV^2}{12\Delta_1}$

Energy between layers and screen (Δ_2) $= \dfrac{KV^2}{48\Delta_2}$

Energy between layers and screen (Δ_3) $= \dfrac{7KV^2}{48\Delta_3}$

$$\therefore \quad C = \frac{K}{6}\left[\frac{1}{\Delta_1} + \frac{1}{4\Delta_2} + \frac{7}{4\Delta_3}\right]$$

or $$C = \left[\frac{4}{\Delta_1} + \frac{1}{\Delta_2} + \frac{7}{\Delta_3}\right]\frac{C_1\Delta}{48}$$

Thus $$C = \frac{C_1\Delta}{48}\left[\frac{4}{\Delta_1} + \frac{1}{\Delta_2} + \frac{7}{\Delta - \Delta_1 - \Delta_2}\right]$$

since $\Delta = \Delta_1 + \Delta_2 + \Delta_3$.

C will be a minimum when $\frac{\partial C}{\partial \Delta_1} = \frac{\partial C}{\partial \Delta_2} = 0$

provided that $\frac{\partial^2 C}{\partial \Delta_1^2} \cdot \frac{\partial^2 C}{\partial \Delta_2^2} > \frac{\partial^2 C}{\partial \Delta_1 \partial \Delta_2}$

Then $$\frac{\partial C}{\partial \Delta_1} = -\frac{4}{\Delta_1^2} + \frac{7}{(\Delta - \Delta_1 - \Delta_2)^2} = 0$$

and $$\frac{\partial C}{\partial \Delta_2} = -\frac{1}{\Delta_2^2} + \frac{7}{(\Delta - \Delta_1 - \Delta_2)^2} = 0$$

$$\therefore \quad \Delta_2 = \frac{\Delta_1}{2} \text{ and } \frac{\Delta_3}{\Delta_1} = \frac{\sqrt{7}}{2}$$

also $$\Delta_3 = \Delta - \frac{3\Delta_1}{2}$$

$$\therefore \quad \Delta = \Delta_1 \ (\sqrt{7}/2 + 3/2)$$
$$\therefore \quad \Delta_1 = 0.354\Delta$$
$$\Delta_2 = 0.177\Delta$$
$$\Delta_3 = 0.467\Delta$$
$$\therefore \quad C_{\min} = 0.665C_1$$

Case D. Fig. 8(*d*).

Three-layer winding.

Energy stored between layers (1) and (2)
$$= \frac{KV^2}{9\Delta_3}$$

Energy stored between layers (2) and (3)
$$= \frac{4KV^2}{9\Delta_2}$$

Energy stored between layer (1) and screen
$$= \frac{KV^2}{108\Delta_4}$$

Energy stored between layer (3) and screens
$$= \frac{19KV^2}{108}$$

$$\text{Total energy} = \frac{KV^2}{18}\left[\frac{19}{6\Delta_1} + \frac{8}{\Delta_2} + \frac{2}{\Delta_3} + \frac{1}{6\Delta_4}\right]$$

$$\therefore \quad C = \frac{\Delta C_1}{18}\left[\frac{19}{6\Delta_1} + \frac{8}{\Delta_2} + \frac{2}{\Delta_3} + \frac{1}{6\Delta_4}\right]$$

If $\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = \frac{\Delta}{4}$

then $$C = \frac{80}{27}C_1$$

8. Conclusion

A number of winding arrangements have been examined and compared by a method which can be applied to any arrangement where the turns of a winding can be regarded as perfectly coupled. This condition is usually applicable within the working band of a transformer but at very high frequencies a further analysis is necessary in order to predict the performance of the transformer.

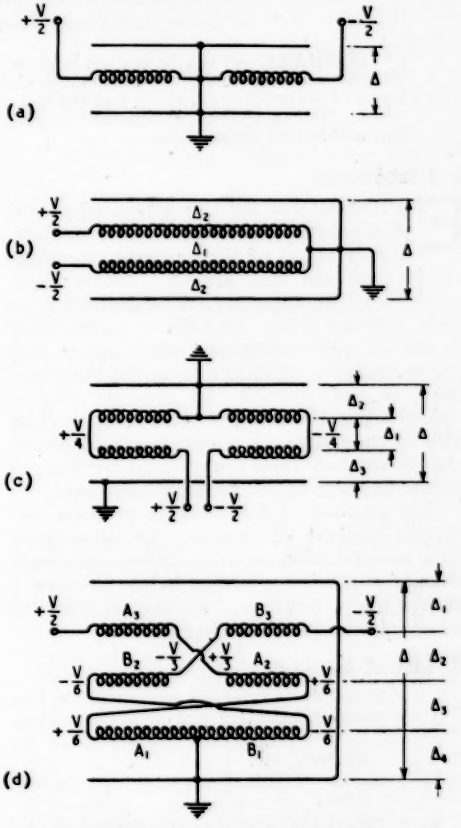

Fig. 8. *The capacitance of totally-screened balanced windings: (a) single-layer winding; (b) two-layer winding; (c) two-layer winding; (d) three-layer winding.*

PROPAGATION CHARACTERISTICS OF A UNIFORM LINE*

By I. F. Macdiarmid, A.M.I.E.E., and H. J. Orchard, Assoc. Brit. I.R.E.

(*Engineer-in-Chief's Training School, Post Office Engineering Department*)

SUMMARY.—A simple method is given of developing expressions which appear to be hitherto unpublished for the attenuation and phase-change coefficients of a uniform line in terms of the primary constants and the real parts of the characteristic impedance and admittance. The expressions are thought to give a clearer physical picture than is provided by the well-known expressions.

1. Introduction

EXPRESSIONS for the attenuation and phase-change coefficients of a uniform line in terms of the primary constants of the line are well known. While these expressions, or approximations derived from their general form, provide a satisfactory means of calculating the attenuation and phase-change coefficients in particular cases, they do not provide the student who is commencing the subject with a very clear idea of the causes of attenuation and phase-change and how these are affected by changes in one or more of the primary constants.

The purpose of this note is to show one simple method of deriving an alternative form of expression for attenuation and phase-change coefficients which helps to give a clearer understanding of the factors affecting attenuation and phase-change.

2. List of Symbols

α = attenuation coefficient of the uniform line.
β = phase-change coefficient of the uniform line.
$\gamma = \alpha + j\beta$ = propagation coefficient of the uniform line.
Z_0 = characteristic impedance of the uniform line.
$Y_0 = 1/Z_0$ = characteristic admittance of the uniform line.
ψ = angle of Z_0.
R_0 = real part of Z_0.
G_0 = real part of Y_0.
R = resistance per unit length of the line (ohms).
L = inductance per unit length of the line (henries).
C = capacitance per unit length of the line (farads).
G = leakance per unit length of the line (mhos).
$Z = R + j\omega L = |Z|\,|\theta$ = total series impedance per unit length.
$Y = G + j\omega C = |Y|\,|\phi$ = total shunt admittance per unit length.

* MS. accepted by the Editor, November 1945.

3. The Propagation Coefficient

Considering a very short length of a uniform line, δx units long, with a single-frequency alternating current being propagated under reflectionless conditions, as in Fig. 1, and using the notation of this figure, we have :—

$$V_1/I_1 = V_2/I_2 = V/I = Z_0 \quad .. \quad (1)$$

and $$I_1/V_1 = I_2/V_2 = I/V = Y_0 \quad .. \quad (2)$$

The decrease in voltage over the length $\delta x \approx IZ\delta x$.

$$\therefore \quad \delta V \approx -IZ\delta x$$

whence $$\partial V/\partial x = -IZ \quad .. \quad .. \quad (3)$$

and similarly

$$\partial I/\partial x = -VY \quad .. \quad .. \quad (4)$$

But $$V = V_0\epsilon^{-\gamma x} \quad .. \quad .. \quad .. \quad (5)$$

where V_0 is the voltage at the sending end of the line and x is the distance from the sending end to the element.

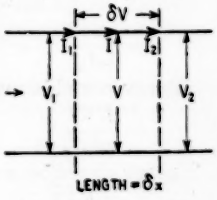

Fig. 1. *Section of a uniform line.*

$\therefore$ by differentiating (5)

$$\partial V/\partial x = -\gamma V$$

and similarly

$$\partial I/\partial x = -\gamma I$$

substituting this in (3) and (4)

$$\gamma = \frac{Z}{V/I} \quad \text{and} \quad \gamma = \frac{Y}{I/V} \quad .. \quad (6)$$

But $V/I = Z_0$ and $I/V = Y_0$

$$\therefore \quad \gamma = Z/Z_0 = Y/Y_0 \quad .. \quad .. \quad (7)$$

Therefore since $Y_0 = 1/Z_0$

$$Z_0 = \sqrt{Z/Y} \quad .. \quad .. \quad .. \quad (8)$$

Therefore from (7) and (8)

$$\gamma = \sqrt{ZY} \quad .. \quad .. \quad .. \quad .. \quad (9)$$

Which is the well known relationship.

Expression (7) can obviously be obtained directly from equations (8) and (9) if the latter are assumed to be known, but the direct method of obtaining (7) given above has been included for the sake of completeness.

3.1. Discussion of Propagation Coefficient

The quantities appearing in equation (7) can all be represented by vectors in a complex plane. Vectors representing the directions of

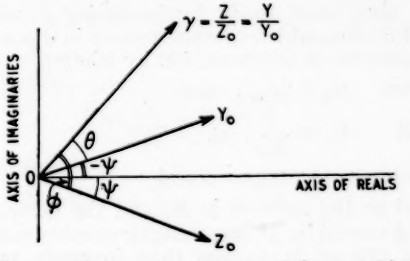

Fig. 2. *Vector representation of the quantities of Equation* (7).

γ, Z_0 and Y_0 are drawn in Fig. 2, where the lengths have no particular significance.

Since $Y_0 = 1/Z_0$

the angle of $Y_0 = -$ (the angle of Z_0).

From (7)

(the angle θ of Z) = (the angle of γ) + (the angle of ψ of Z_0)

and

(the angle of ϕ of Y) = (the angle of γ) + (the angle $-\psi$ of Y_0)

The above angles are all indicated in Fig. 2.

4. The Attenuation and Phase-Change Coefficients

The information contained in Fig. 2 can be used to find the values of the real and imaginary parts of γ, i.e. α and β. To do this the vector representing γ must be drawn to scale so that its length represents the magnitude of γ. This is done in Fig. 3 where the figure is completed by drawing perpendiculars to the Z_0 and Y_0 vectors and to the axis of reals, as shown in the figure.

Using the notation shown in Fig. 3,

$$\angle EOB = \angle BOH = \psi$$

Whence and from the geometry of the figure

$$\angle GAB = \angle BAD = \psi$$

$$\therefore \quad HB = BE \qquad .. \quad .. \quad .. \quad (10)$$

$$\text{and} \quad GB = BD \qquad .. \quad .. \quad .. \quad (11)$$

Since OA represents γ and OB is the real part of OA

$$\alpha = OB = \tfrac{1}{2}(OG + OD) \quad .. \quad (12)$$

$$\text{Similarly } \beta = AB = \tfrac{1}{2}(AH + AE) \quad .. \quad (13)$$

$$\text{Since} \quad OG = OF \cdot \frac{1}{\cos\psi}$$

$$\text{and} \quad OF = AO \cdot \cos\phi$$

$$= \frac{|Y|}{|Y_0|} \cdot \cos\phi \quad \text{because } |\gamma| = \frac{|Y|}{|Y_0|}$$

$$\therefore \quad OG = \frac{|Y|\cos\phi}{|Y_0|\cos\psi}$$

$$= G/G_0 \quad \text{as } \cos\psi = \cos(-\psi)$$

$$.. \quad .. \quad (14)$$

$$\text{Since} \quad OD = OC \cdot \frac{1}{\cos\psi}$$

$$\text{and} \quad OC = OA \cdot \cos\theta$$

$$= \frac{|Z|}{|Z_0|} \cdot \cos\theta \quad \text{because } |\gamma| = \frac{|Z|}{|Z_0|}$$

$$\therefore \quad OD = \frac{|Z|\cos\theta}{|Z_0|\cos\psi}$$

$$= R/R_0 \quad .. \quad .. \quad .. \quad (15)$$

Therefore from (12), (14) and (15)

$$\alpha = \tfrac{1}{2}\left(\frac{R}{R_0} + \frac{G}{G_0}\right) \quad .. \quad .. \quad (16)$$

In the same way it can be shown from (13) that

$$\beta = \tfrac{1}{2}\left(\frac{\omega L}{R_0} + \frac{\omega C}{G_0}\right) \quad .. \quad .. \quad (17)$$

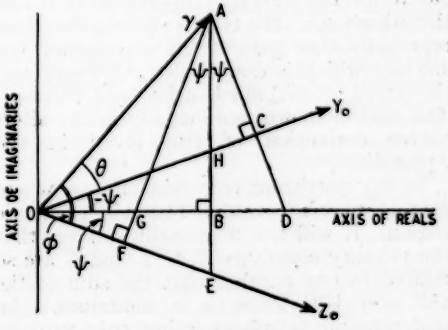

Fig. 3. *Vectorial derivation of* $\alpha = OB$ *and* $\beta = AB$.

Approximate expressions, having some relationship with (16) have been given by W. L. Everitt[1] for two special cases, viz.:

For unloaded cables $\alpha \approx 0.707 \dfrac{R}{|Z_0|}$

For loaded cables $\alpha \approx \frac{1}{2}\left(\frac{R}{Z_0} + \frac{GZ_0}{1}\right)$

These expressions are derived independently, from the two usual approximations for unloaded cables and loaded cables at voice frequencies (Equations (19) and (21) in this note). Being derived from approximations they only hold numerically where the approximations apply. The expression given in (16) being derived without any approximation is, however, exact under all circumstances.

4.1. Discussion of Attenuation Coefficient

Examination of (16) and (17) brings out a number of interesting points about the behaviour of lines. For example the attenuation of a line of given characteristic impedance depends solely on the resistive parts of Z and Y, whereas the phase change is determined by the reactive parts of Z and Y. Frequency will affect the attenuation of a line only in so far as it affects the real parts of the characteristic impedance and characteristic admittance, unless, as happens at high frequencies, it also alters the values of R and G.

The attenuation coefficient of a line depends upon the fraction of the transmitted power which is dissipated in the line and appears as heat in the conductors and the dielectric; that is upon the I^2R and V^2G losses. Now the expression for the attenuation coefficient given by (16) is composed of two terms, one directly proportional to R and the other to G. The term $\frac{1}{2} \cdot (R/R_0)$ therefore represents that part of the attenuation coefficient which is due to the I^2R losses and the term $\frac{1}{2} \cdot (G/G_0)$ likewise for the V^2G losses. This fact is shown more clearly in the alternative derivation of (16) given in the Appendix.

In any particular transmission line, where R and G may be considered fixed by economic reasons, it will not necessarily happen that the primary constants R, C, L and G are so related to one another that the sum of the I^2R and V^2G losses is a minimum. An underground telephone cable pair used at voice frequencies provides one such example. Here L and C are so related to R and G that the I^2R losses are much greater than the V^2G losses (i.e. $\frac{1}{2} \cdot (R/R_0)$ is much greater than $\frac{1}{2} \cdot (G/G_0)$), and also that Z_0 has a negative angle of approximately 45° (i.e. $\psi = -45°$), as will be shown in Section 5.1.

If now some means can be found of increasing R_0 then the I^2R losses (i.e. $\frac{1}{2} \cdot (R/R_0)$) will be reduced, resulting in a reduction in the attenuation coefficient. Such an increase in R_0 may not be possible without a proportionate decrease in G_0, but the net effect would still be a reduction in the attenuation coefficient up to the point where the I^2R and V^2G losses became equal.

R_0 can be increased either by decreasing ψ or by increasing $|Z_0|$. Both of these can be effected by increasing the ratio L/C. This can be done most easily by increasing L with inductance added continuously or in discrete quantities at intervals, i.e. by loading.

Since $R_0 = |Z_0| \cdot \cos\psi$

and $G_0 = \frac{1}{|Z_0|} \cdot \cos(-\psi)$

we have $R_0 \cdot G_0 = \cos^2\psi$

and so the increase in R_0 and the decrease in ψ caused by adding inductance must make the change in G_0 less than inversely proportional to the change in R_0.

Hence adding inductance will make the attenuation coefficient smaller within the limitations given above.

5. Approximations for the Attenuation and Phase-Change Coefficients

The usual well known approximations for the attenuation and phase-change coefficient can be derived readily from (16) and (17).

5.1. *Unloaded Cable at Voice Frequencies*

At the frequencies under consideration, usually

$$R \gg \omega L \quad \text{and} \quad \omega C \gg G$$

Then $$Z_0 = \sqrt{\frac{R}{j\omega C}} = \sqrt{\frac{R}{\omega C}}\,\angle 45°$$

whence

$$R_0 = \sqrt{\frac{R}{2\omega C}} \quad \text{and} \quad G_0 = \sqrt{\frac{\omega C}{2R}} \qquad (18)$$

$\therefore$ From (16)

$$\alpha = \tfrac{1}{2}\left\{\sqrt{2\omega CR} + G \cdot \sqrt{\frac{2R}{\omega C}}\right\}$$

As G is very small the second term is normally negligible.

$$\therefore \quad \alpha \approx \sqrt{\frac{\omega CR}{2}} \qquad .. \qquad .. \qquad .. \qquad (19)$$

and from (17) and (18)

$$\beta = \tfrac{1}{2}\left\{\frac{\omega L}{1} \cdot \sqrt{\frac{2\omega C}{R}} + \sqrt{2\omega CR}\right\}$$

and since ωL is very small

$$\beta \approx \sqrt{\frac{\omega CR}{2}} \quad .. \quad .. \quad .. \quad (20)$$

5.2. *Loaded Uniform Cable and Unloaded Cable at High Frequencies*

At frequencies such that $\omega L \gg R$ and $\omega C \gg G$ it can be shown[2] that

$$Z_0 \approx \sqrt{\frac{L}{C}} \left| \underline{\tfrac{1}{2}\,(G/\omega C - R/\omega L)} \right.$$

The angle is normally only a few degrees

$\therefore \quad R_0 \approx \sqrt{L/C}$ and $G_0 \approx \sqrt{C/L}$

$\therefore$ From (16)

$$\alpha \approx \frac{R}{2}\sqrt{C/L} + \frac{G}{2}\sqrt{L/C} \quad .. \quad (21)$$

and from (17)

$$\beta \approx \tfrac{1}{2}\{\omega\sqrt{L\,.\,C} + \omega\sqrt{L\,.\,C}\}$$
$$\approx \omega\sqrt{L\,.\,C} \quad .. \quad .. \quad .. \quad (22)$$

Conclusions

Expressions which are believed to have been hitherto unpublished have been derived for the attenuation and phase-change coefficients of a uniform line. They are :—

$$\alpha = \tfrac{1}{2}\,(R/R_0 + G/G_0)$$
$$\beta = \tfrac{1}{2}\,(\omega L/R_0 + \omega C/G_0)$$

The method of deriving the expressions which has been given is not the only one possible but has been chosen because it yields the expressions for both α and β, and can readily be followed by a student with limited mathematical equipment. The latter is generally advantageous as it permits the student to concentrate on the physics of the problem rather than the mathematics. An alternative method of obtaining the expression for α which is equally simple, if not simpler than that already used, is given in the Appendix. This method, however, cannot be readily used to develop the expression for β.

The main advantages of the expressions developed above appear to be in the teaching of principles of transmission lines, since they do not seem to have any advantage over the conventional expressions for calculating numerical values of attenuation and phase-change coefficients.

APPENDIX

Alternative Derivation of Equation (16)

The power entering the element of line in Fig. 1 is $P_1 = V_1\,.\,I_1 \cos\psi$ where V_1 and I_1 are R.M.S. values of voltage and current respectively and since the length of the element is approaching zero, the power leaving the element is

$$P_2 \approx P_1 - I_1^2\,.\,R\,.\,\delta x - V_1^2\,.\,G\,.\,\delta x$$

$\therefore$ Power Ratio $P_2/P_1 = \epsilon^{-2\alpha\,\delta x}$

$$\approx \frac{V_1\,.\,I_1\cos\psi - (I_1^2R + V_1^2G)\delta x}{V_1\,.\,I_1\,.\,\cos\psi}$$
$$\approx 1 - \frac{(I_1^2R + V_1^2G)\delta x}{V_1\,.\,I_1\cos\psi} \quad .. \quad (23)$$

But $\epsilon^{-2\alpha\,\delta x} = 1 - \frac{2\alpha\delta x}{1!} + \frac{(2\alpha\delta x)^2}{2!} - \frac{(2\alpha\delta x)^3}{3!} + \ldots$

$\therefore$ From (23) when $\delta x \to 0$

$$2\alpha\delta x = \frac{(I_1^2R + V_1^2G)\delta x}{V_1\,.\,I_1\cos\psi}$$

$$\therefore \quad 2\alpha = \frac{R}{V_1/I_1\,.\,\cos\psi} + \frac{G}{I_1/V_1\,.\,\cos\psi}$$

$$\therefore \quad \alpha = \tfrac{1}{2}\{R/R_0 + G/G_0\} \quad \text{since } V_1/I_1 = |Z_0|$$

REFERENCES

[1] W. L. Everitt, "Communication Engineering," second edition, 1937. (McGraw-Hill Book Co.) p. 116 and p. 119.

[2] E.g., see Ref. 1, p. 118.

Book Review

Problèms de propagations guidées des ondes électromagnétiques.

By Louis de Broglie. Pp. 114 and 14 Figs. Gauthier-Villars, 55, Quai des Grands-Augustins, Paris. 160 francs.

This is a monograph by a well-known French scientist who is Professor of Physics at Paris. It was published in 1941. De Brogle's work on " La Mécanique Ondulatoire des Systèmes de Corpuscules " is well known and we were somewhat surprised to find that he had become interested in waveguides. He says in the preface that he was attracted by the great theoretical and practical importance of the transmission of ultra short waves in tubes and horns and was led to study the published works on the subject, particularly those of Clavier and Brillouin. He claims no originality, but has merely written a résumé of these works in a form which he hopes will be helpful to physicists and radio-engineers. It is divided into six chapters, the first of which deals with the Maxwell equations and the Hertzian vector potentials. In the following chapters the formulae are developed for the various types of wave-guides and for the various modes of propagation, resonance conditions in cavities, loss of energy in the walls and the resulting attenuation. Chapter V deals with the propagation in horns, i.e. where the cross-section of the guide is gradually increasing, and Chapter VI with the diffraction of the waves at the mouth of the guide or horn. There is a bibliography of eighteen items, giving references to the papers by Barrow, Southworth, and others.

The work is, of course, rather mathematical, as it must be, but it is developed step by step and all set out very clearly. It is a carefully prepared review of the subject written by a theoretical physicist of world-wide repute.

G. W. O. H.

IRON-CORED LOOP RECEIVING AERIAL*

By R. E. Burgess, B.Sc.

(Communication from the National Physical Laboratory)

SUMMARY.—The complex effective permeability of a mass core is expressed in terms of the relevant factors, and the imaginary part is related to the eddy current loss in the particles, which should predominate over other components of loss.

The increase of pick-up due to a spheroidal core is calculated and it is shown that the core should be elongated in a direction parallel to the axis of the loop. The effect of a hollow spheroidal core is discussed and it is found that in a typical case 80 per cent. of the iron can be removed before the increase of pick-up is halved ; the effect of spacing the winding from the core is treated approximately.

Recommendations are made regarding the design for maximum sensitivity.

1. Introduction

IT is well known that the insertion of a ferromagnetic core into a coil increases its self-inductance by virtue of the increase of flux. In this case the field producing the flux is due to the current in the coil, but if it were due to an external source an increase of flux would still be obtained. Thus, the use of an iron core to increase the pick-up of a loop is at once suggested. At radio frequencies the losses in a solid core would be prohibitive, and it is necessary to use a mass core consisting of finely-divided iron particles, insulated from each other and compressed into a suitable form.

Apart from increasing the signal pick-up of the loop, the core will produce an increase of inductance and effective resistance, and thus it is necessary to examine to what extent the latter effect offsets the increased pick-up. There appears to be no previously published work on this problem, and the object of the present paper is to discuss these considerations in detail.

The object of an iron-cored loop is to increase the sensitivity for a given size, or conversely, to reduce the size for a given sensitivity. Thus it has particular application where space is limited, e.g. on aircraft where the greater weight of the iron-cored loop will be offset, at least partially, by the reduced drag for the smaller size. Such an iron-cored loop has already been used in aircraft and W. J. Polydoroff has been granted a patent for this invention (British Patent 522,492).

* MS. accepted by the Editor, February 1946.

2. The Magnetic Properties of a Mass Core

If the mass core contains uniform spherical particles of iron of permeability μ_0 and conductivity σ (e.m.u.) insulated from each other, the effective permeability of the core is given[1, 2, 5] by

$$\mu = \frac{(\mu_0 + 2) + 2p\,(\mu_0 - 1)}{(\mu_0 + 2) - p\,(\mu_0 - 1)} = \frac{1 + 2pM_0}{1 - pM_0} \quad \ldots \quad (1)$$

where p is the fractional volume of the iron and $M_0 = (\mu_0 - 1)/(\mu_0 + 2)$. This equation can also be written in the form

$$\frac{\mu - 1}{\mu + 2} = p\frac{\mu_0 - 1}{\mu_0 + 2} = pM_0 \quad \ldots \quad (2)$$

showing the additive nature of the term $(\mu_0 - 1)/(\mu_0 + 2)$ since the corresponding term for the insulator is zero.

The value of p for a cubical arrangement of spheres in contact is $\pi/6 = 0.52$ while for the most closely packed formation it is $\sqrt{2}\pi/6 = 0.74$. Thus in cores having a higher value of p than 0.74 the particles must be non-spherical and in general their irregularities will be randomly distributed in direction, so that the core is still magnetically isotropic. When the iron content is high ($p > 0.9$) the equation (1) based on simple statistical considerations is no longer accurate and permeabilities some 50 per cent. higher than those given by this equation have been observed.[5]

The relation between μ and p for $\mu_0 = 50$, 100 and infinity is shown in Fig. 1. It is seen that as the permeability of the iron

particles is increased indefinitely, the effective permeability of the core tends to an upper limit given by

$$\mu_{max} = \frac{1 + 2p}{1 - p} \quad .. \quad .. \quad .. \quad (3)$$

on account of a lower limit to the magnetic reluctance being set by the non-magnetic gaps between the particles.

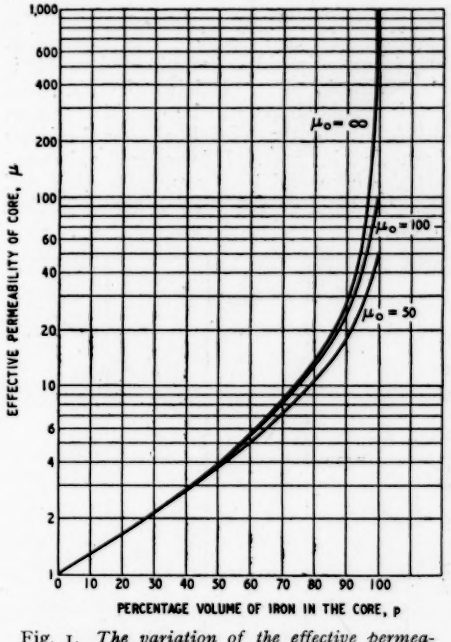

Fig. 1. *The variation of the effective permeability of the core with the volume proportions for various particle permeabilities.*

The equations (1), (2) and (3) are only strictly valid for magnetostatic fields. In alternating fields, eddy currents are set up in the particles which introduce a power loss depending upon the particle size and frequency, and the effective permeability of the core assumes a complex value $(\mu - j\mu')$. A parameter α can be defined[3] which determines the behaviour of the μ and μ' with frequency:

$$\alpha = r\sqrt{2\pi\omega\sigma\mu_0} \quad .. \quad .. \quad .. \quad (4)$$

where r is the particle radius, and it may be said that below the "bounding frequency" f_g at which $\alpha = 1$ the effect of the eddy currents on the real part of the permeability is small. Thus

$$f_g = \frac{1}{4\pi^2\mu_0\sigma r^2} \quad .. \quad .. \quad .. \quad (5)$$

For example if $\mu_0 = 100$, $\sigma = 10^{-4}$ e.m.u. (for iron) and $r = 5 \cdot 10^{-4}$ cm it is found that $f_g = 10$ Mc/s.

From Divilkovsky's analysis[3] it may be shown that in order to take account of the eddy currents in the particles the expression $M_0 = (\mu_0 - 1)/(\mu_0 + 2)$ in Equation (2) should be replaced by

$$M = \left[\frac{\mu_0 - 1}{\mu + 2} - \frac{3}{175}\frac{\mu_0(\mu_0 + 9)}{(\mu_0 + 2)^3}\alpha^4\right] - j\frac{3}{5}\frac{\mu_0}{(\mu_0 + 2)^2}\alpha^2 \quad .. \quad (6)$$

when $\alpha^2 \ll 1$, that is at frequencies well below the bounding frequency which is the condition in practical iron-cored systems.

On making this substitution it is found that the imaginary part of the permeability is given by

$$\mu' = \frac{18\pi}{5}\frac{p\mu_0^2\,\omega r^2\sigma}{[(\mu_0 + 2) - p(\mu_0 - 1)]^2} \quad .. \quad (7)$$

Eddy-current loss in the core is the most important, and in a well-designed iron-cored loop would represent the major part of the loss due to the core.

In the case where the core completely fills the loop without magnetic leakage, the inductance is increased μ-fold from L_0 to L while the eddy-current loss resistance R_e is related to the imaginary component μ' of the effective permeability:—

$$R_e + j\omega L = j\omega L_0(\mu - j\mu') \quad .. \quad (8)$$

or

$$R_e = \omega L_0\mu' = \omega L\,\mu'/\mu \quad .. \quad (9)$$

In the case where magnetic leakage exists, that is $L < \mu L_0$, which in practice always holds, it is *not* correct to write

$$R_e + j\omega L = j\omega L_0 k(\mu - j\mu')$$

which implies that $R_e/\omega L$ always equals μ'/μ.

In Appendix I the relation between the increase of resistance and inductance of a solenoid due to a spheroidal core is deduced. It is shown that whatever the relative size of the core and coil

$$\frac{R_e}{\omega(L - L_0)} = \frac{\mu'}{(\mu - 1)\,[(\mu - 1)g + 1]} \quad .. \quad .. \quad (10)$$

where g is the demagnetization coefficient of the spheroid. In the case of an infinitely elongated core ($g = 0$) this equation agrees with (9) but in other cases the latter does not hold.

It appears from the equations developed

in Appendix I that there is no general equivalent circuit for an iron-cored coil that will represent its impedance irrespective of the configuration of core and coil. It may, however, be shown that if there is no magnetic leakage, i.e. the inductance is increased μ-fold by the core, then the " eddy current coefficient "[4] defined as R_e/f^2L is given by

$$W \equiv \frac{R_e}{f^2L} = \frac{8\pi^3}{5}\sigma r^2\mu \quad .. \quad .. \quad (11)$$

Thus the eddy current loss can be reduced either (a) by very fine subdivision of the particles or (b) by reducing the conductivity of the particles, which is possible by the use of certain nickel-iron alloys. In practice the first method is the most usual, the particles then being of pure iron.

Just as Equ. (1) for the permeability gives too low a figure for materials of large iron content ($\phi > 0.9$) so Equ. (11) tends to underestimate the eddy current losses owing to proximity losses and the increased tendency for electrical contact between the particles.

3. Increase of Pick-up due to the Core

Let the core have effective permeability μ and let the loop be wound directly on the surface of the core. The increase of pick-up due to the core is then equal to the ratio of the flux in the core to that when the core is absent. This increase will not in general be μ-fold since the induced magnetization gives rise to a secondary field which is in opposition to the original field.

If the original field H_0 is uniform and the induced magnetization is I, then the secondary field is written as $-4\pi gI$ where g is termed the demagnetization coefficient and is a function of the shape of the core and its orientation with respect to the field.

The effective magnetizing force in the core is thus

$$H = H_0 - 4\pi gI$$

although it is clear that these must be statistical values in the case of a mass core on account of the discrete magnetic and non-magnetic regions.

Now by definition of μ

$$I = KH = \frac{\mu - 1}{4\pi} H$$

where K is the susceptibility, and thus

$$H = \frac{H_0}{1 + g(\mu - 1)} \quad .. \quad .. \quad (12)$$

The flux density in the core is

$$B = \mu H = \frac{\mu}{1 + g(\mu - 1)} H_0$$

so the step-up in flux and hence of e.m.f. due to the core is

$$m \equiv \frac{B}{H_0} = \frac{\mu}{1 + g(\mu - 1)} \quad .. \quad .. \quad (13)$$

The only general shape for which the induced magnetization is uniform when H_0 is uniform is an ellipsoid; the expression for g of an ellipsoid is complicated unless it is an ellipsoid of revolution (spheroid), and therefore this shape will be considered now, for it is sufficiently general to cover the forms of practical interest. It is naturally assumed that the core and loop rotate together and that the usual cosine polar diagram is thus obtained.

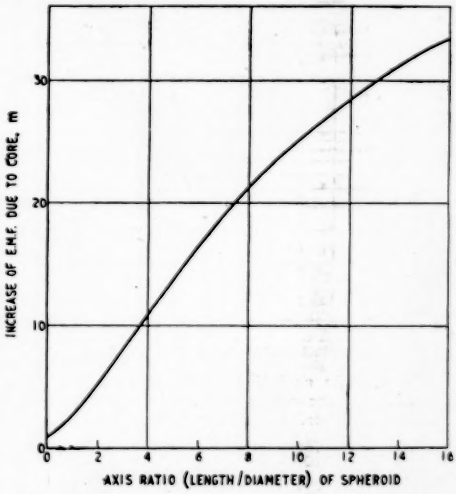

Fig. 2. *Increase of e.m.f. in a loop due to the insertion of a spheroid of permeability* 50.

It is seen from Equ. (13) that g should be as small as possible, which means that the core should be elongated in the direction of the field so that the " poles " are removed as far as possible from the centre. For this reason we will consider the case of a prolate spheroid with its major axis parallel to the field, for which

$$g = \frac{1 - e^2}{e^2}\left(\frac{1}{2e}\log_e\frac{1 + e}{1 - e} - 1\right) \quad .. \quad (14)$$

where e is the eccentricity which is related to the axis ratio a/b by

$$e = \sqrt{1 - \frac{b^2}{a^2}} \qquad .. \quad .. \quad .. \quad (15)$$

In the case of the sphere for which $e = 0$ we find that $g = \frac{1}{3}$. To illustrate these equations Fig. 2 shows the variation of the step-up m with the axis ratio a/b when the core has a permeability of 50.

It is important to note that in general the increase of e.m.f. and the increase of inductance due to the core are not the same (as is assumed in the Patent mentioned earlier). These must both be smaller than μ, and only when the coil produces a uniform field (as in a long solenoid) will they be equal.

It is of interest to examine the effect of hollowing the core on the increase of e.m.f. which it gives. In Appendix II the case of a hollow spheroid is analysed, and it is found that the step-up m does not fall appreciably below the value for a solid spheroid until the shell of magnetic material is quite thin. For example Fig. 3 shows the variation of m with the ratio γ of the cross-sectional area of the hollow space to the total cross section for a permeability of 50 and an external axis ratio of 5 : 1. It is seen that γ may be as large as 0.8 before m is reduced to half the value for the solid spheroid.

The analysis shows that a hollow spheroid gives the same step-up m as the solid spheroid of the same outer dimensions but smaller permeability μ_e, that is

$$m = \frac{\mu_e}{1 + g_2(\mu_e - 1)} \qquad .. \quad .. \quad (16)$$

where g_2 is the demagnetization coefficient for the outer surface. The " equivalent permeability " μ_e is given to a close approximation by the mean of the product (area $\times$ permeability) for the cross section, viz. :

$$\mu_e \approx \gamma + (1 - \gamma)\mu \qquad .. \quad .. \quad (17)$$

The advantage of hollowing the core is clearly to save weight and material and the extent to which it is carried out is determined by the desirable compromise between these factors and the step-up m which governs the sensitivity.

If the core is of high permeability the field external to it and in its immediate vicinity will be considerably smaller than the original field H_0 and thus if the loop is spaced from the core, as will probably be necessary to avoid excessive dielectric loss, the step-up will be reduced to a value given approximately by

$$m' = m\frac{A_0}{A} \qquad .. \quad .. \quad .. \quad (18)$$

where A_0 is the cross-sectional area of the core and A is that of the loop.

By way of illustration we may take a fairly typical case corresponding approximately to that of the loop used in German aircraft which was described by C. P. Edwards of the Royal Aircraft Establishment[6].

With core permeability $\mu = 50$

and ratio of radii $\frac{a}{b} = 4$

then the demagnetization coefficient $g = 0.075$ giving for the increase of pick-up $m = 10.6$.

If, however, the core is hollow with the hollow area being 40 per cent. of the whole area at the central cross section

$$\gamma = 0.4$$

giving $\mu_e = 34$

for which the increase of pick-up will now be $m = 10.0$, showing that the hollowing produces but little reduction of step-up.

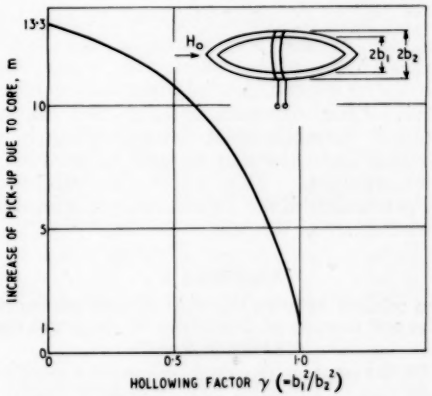

Fig. 3. *Increase of pick-up for a hollow spheroidal core of external axis ratio* 5 : 1 *and permeability* 50.

If the loop is spaced from the core so that the latter's outer cross-sectional area occupies only 60 per cent. of the loop area, the resultant step-up with the hollow core will be

$$m' = 0.6\ m = 6.0.$$

4. Conclusions

The main features of the iron-cored loop have been treated, and it may be concluded:—

(a) the core should have a high permeability ;

(b) the core should be elongated in the direction of the axis of the loop ;

(c) the particle size should be as small as possible to minimize W ;

(d) the core can be hollow to save weight and material and although this reduces the step-up m, a hollowing ratio of about 50 per cent. could usually be adopted with advantage ;

(e) the loop should be spaced from the core just sufficiently to prevent excessive dielectric loss, remembering that the step-up will be reduced ;

(f) the loop conductor should be designed to have minimum copper losses *with the core inserted* ; since the proximity loss will usually be relatively small owing to the reduced magnetic field at the conductor, the main consideration is to minimize the skin effect, implying that stranded wire will usually be advantageous.

5. Acknowledgements

The work described above was conducted as part of the programme of the Radio Research Board to whom this paper was circulated as a confidential paper in November 1941. The co-operation of the Royal Aircraft Establishment in providing loop systems from German aircraft is gratefully acknowledged. This paper is published by permission of the Department of Scientific and Industrial Research.

APPENDIX I

The relation between the eddy current loss resistance and increase of inductance of a solenoid due to a spheroidal core.

Divilkovsky[3] has discussed the case of a spherical mass core in a solenoid and the analysis will now be extended to the case of a spheroidal core. The solenoid is assumed to be sufficiently long for the field within it to be substantially uniform (H_0) in the absence of the core.

If the demagnetization coefficient of the spheroid is g the mean (or macroscopic) field within the core will be given by

$$H = \frac{H_0}{1 + g\,(\mu - j\mu' - 1)}$$

where $(\mu - j\mu')$ is the complex effective permeability of the core for the frequency concerned.

Now it may be shown that the field in which each particle is situated, i.e. the field observed when the particle is removed, is

$$H' = \frac{H}{1 - pM}$$

where M is the complex effective value of $(\mu_e - 1)/(\mu_0 + 2)$ defined by equation (6). Bearing in mind that

$$\mu - j\mu' = \frac{1 + 2pM}{1 - pM}$$

it is found that

$$H' = \frac{H_0}{1 - pM\,(1 - 3g)}$$

Thus only for the simple case of a sphere in which $g = 1/3$ will $H' = H_0$.

Knowing the field H' at the particles, their magnetic moment can be calculated and the value of the magnetic vector potential at the solenoid winding found. From this, the electric intensity is deduced and thus the e.m.f. due to the core is found ; the component of e.m.f. in phase with the current gives the eddy current loss resistance while the quadrature component gives the increase of inductance.

In the present case of a spheroid it is finally found that

$$R_e + j\omega L = j\omega L_0\left[1 + \frac{3pM}{1 - pM\,(1 - 3g)}\,\frac{V_0}{V}\right]$$

where V_0 and V are the volumes of the core and coil respectively. If we use the relation

$$\frac{\mu - j\mu' - 1}{\mu - j\mu' + 2} = pM$$

we find that

$$R_e + j\omega L = j\omega L_0\left[1 + \frac{\mu - j\mu' - 1}{(\mu - j\mu' - 1)\,g + 1}\,\frac{V_0}{V}\right]$$

giving to a very close approximation as $\mu' \ll \mu$ in practice

$$L = L_0\left[1 + \frac{\mu - 1}{(\mu - 1)\,g + 1}\,\frac{V_0}{V}\right]$$

and

$$\frac{R_e}{\omega L_0} = \frac{\mu'}{[(\mu - 1)\,g + 1]^2} \cdot \frac{V_0}{V}$$

We may thus eliminate $\frac{V_0}{V}$ and obtain a relation which is independent of the relative volumes of the core and coil :

$$\frac{R_e}{\omega\,(L - L_0)} = \frac{\mu'}{(\mu - 1)\,[(\mu - 1)\,g + 1]} \qquad (10)$$

Alternatively we may eliminate g and obtain a relation which is not dependent on the ratio of length to diameter of the core.

$$\frac{R_e L_0}{\omega\,(L - L_0)^2} = \frac{\mu'}{(\mu - 1)^2}\,\frac{V}{V_0}$$

$$= \frac{2\pi}{5}\,\frac{\sigma\omega r^2}{p_0}\left(\frac{\mu_0}{\mu_0 - 1}\right)^2 \frac{V}{V_0}$$

Thus if the ratio of the total volume pV_0 of the iron particles to the volume V of the coil is denoted by p_0 we have in general

$$\frac{R_e L_0}{\omega^2\,(L - L_0)^2}$$

$$= \frac{2\pi}{5}\,\frac{\sigma r^2}{p_0}\left(\frac{\mu_0}{\mu_0 - 1}\right)^2 \approx \frac{2\pi}{5}\,\frac{\sigma r^2}{p_0} \text{ since } \mu_0 \gg 1$$

If the core is long or toroidal ($g = 0$) and fills the coil ($V_0 = V$) we have $L = \mu L_0$ and the "eddy current coefficient" is given by

$$W \equiv \frac{R_e}{f^2 L} = \frac{8\pi^3}{5}\sigma r^2 \frac{(\mu-1)^2}{p\mu}\left(\frac{\mu_0}{\mu_0-1}\right)^2$$
$$\approx \frac{8\pi^3}{5}\sigma r^2 \mu \qquad .. \qquad .. \qquad (11)$$

the latter approximation being very close for $\mu \gg 2$.

APPENDIX II

Increase of flux due to hollow core bounded by confocal prolate spheroidal surfaces.

The form of core which will be investigated is that bounded by confocal prolate spheroidal surfaces since this case gives a fairly simple analysis while it is sufficiently general to be applied to a wide variety of cores and in particular gives useful information regarding the effect of a hollow core.

The loop is assumed to be wound at the central circular cross-section of the core while the field is perpendicular to the plane of the loop, i.e. along the major axis of the core. The total flux threading the loop will be calculated and thus the step-up m due to the core is found ; the results can also be conveniently expressed in terms of the equivalent permeability which a solid spheroid of the same outer dimensions must have to give the same increase m in flux.

Let the surfaces bounding the core have equations

$$x^2/a_1^2 + r^2/b_1^2 = 1 \quad \text{and} \quad x^2/a_2^2 + r^2/b_2^2 = 1$$

where $r^2 = y^2 + z^2$ and $a_2^2 - b_2^2 = a_1^2 - b_1^2$

since the surfaces are confocal.

Let the core material have permeability μ and the original external field be H_0 parallel to the x-axis. The potential of this field is

$$V_0 = -H_0 x$$

Now it is known that the additional potential due to the core has the form

$$V_1 = (c_1 g_1 + c_2 g_2) H_0 x$$

in the hollow interior, while in the shell it has the form $V_2 = (c_1 g + c_2 g_2) H_0 x$

where $g_1 = \dfrac{a_1 b_1^2}{2}\displaystyle\int_0^\infty \frac{d\lambda}{(a_1^2+\lambda)^{3/2}(b_1^2+\lambda)}$

$$= \frac{1-e_1^2}{e_1^2}\left(\frac{1}{2e_1}\log_e\frac{1+e_1}{1-e_1} - 1\right)$$

in which

$$e_1 = \frac{1}{a_1}\sqrt{a_1^2 - b_1^2}$$

is the eccentricity of the inner surface of the core

$$g_2 = \frac{a_2 b_2^2}{2}\int_0^\infty \frac{d\lambda}{(a_2^2+\lambda)^{3/2}(b_2^2+\lambda)}$$

$$= \frac{1-e_2^2}{e_2^2}\left(\frac{1}{2e_2}\log_e\frac{1+e_2}{1-e_2} - 1\right)$$

in which $e_2 = \dfrac{1}{a_2}\sqrt{a_2^2 - b_2^2} = \dfrac{a_1}{a_2}e_1$ is the eccentricity of the outer surface, while

$$g = \frac{a_1 b_1^2}{2}\int_u^\infty \frac{d\lambda}{(a_1^2+\lambda)^{3/2}(b_1^2+\lambda)}$$

$$= e\left(\frac{1-e_1^2}{e_1^3}\right)\left(\frac{1}{2e}\log_e\frac{1+e}{1-e} - 1\right)$$

where $e = \sqrt{\dfrac{a_1^2 - b_1^2}{a_1^2+u}}$ is the eccentricity of the confocal surface corresponding to the parameter u which is the positive root of

$$\frac{x^2}{a_1^2+u} + \frac{r^2}{b_1^2+u} = 1.$$

The constants c_1 and c_2 are determined by the boundary conditions and it is found that

$$-\frac{\mu-1}{c_1}$$
$$= \frac{(\mu-1)[\mu - g_1(\mu-1) - \beta(1-g_2)(\mu-1)]}{c_2}$$
$$= [\mu - g_1(\mu-1)][1+g_2(\mu-1)] - \beta g_2(1-g_2)(\mu-1)^2$$

where $\beta = \dfrac{a_1 b_1^2}{a_2 b_2^2}$

is the ratio of the volume of the hollow to the total volume. It will be recognised that g_1 and g_2 are the demagnetization coefficients for spheroids of eccentricities e_1 and e_2 respectively, cf. equation (14).

The field in the hollow space is uniform and is given by

$$H_{1x} = -\frac{\partial}{\partial x}(V_0 + V_1) = H_0(1 - c_1 g_1 - c_2 g_2)$$

while the x-component of field inside the shell is given by

$$H_{2x} = -\frac{\partial}{\partial x}(V_0 + V_2)$$
$$= H_0\left[1 - c_1\frac{\partial}{\partial x}(gx) - c_2 g_2\right]$$

As the loop is wound around the central circular section of the core, its equation is $x = 0$ and $r = b_2$. Thus the relevant value of H_{2x} is that in the plane $x = 0$:—

$$H_{2x} = H_0(1 - c_1 g - c_2 g_2)$$

which is clearly non-uniform since g is a function of u which in turn is a function of r, namely at $x = 0$

$$r^2 = b_1^2 + u$$

Thus the total magnetic flux perpendicular to the plane of the loop is

$$\Phi = \pi b_1^2 H_{1x} + \mu\int_{b_1}^{b_2} H_{2x} 2\pi r \, . \, dr$$

Hence the step up in flux due to the core is

$$m = \Phi/\pi b_2^2 H_0 = \frac{b_1^2}{b_2^2}\left(1 - c_1 g_1 - c_2 g_2\right)$$
$$+ \mu\left(1 - \frac{b_1^2}{b_2^2}\right)\left(1 - c_2 g_2\right) - \frac{\mu c_1}{b_2^2}\int_{b_1}^{b_2} 2gr \, . \, dr$$

which after some reduction gives

$$m = \frac{\mu}{\mu-1}c_2$$
$$= \mu\frac{\mu - g_1(\mu-1) - \beta(1-g_2)(\mu-1)}{[\mu - g_1(\mu-1)][1+g_2(\mu-1)] - \beta g_2(1-g_2)(\mu-1)^2}$$

In the limiting case of a solid spheroid ($\beta = 0$, $g_1 = 0$) we have as an upper limit to m the familiar expression

$$m = \frac{\mu}{1 + g_2(\mu-1)}$$

while for an infinitely thin shell ($\beta = 1$, $g_1 = g_2$) the lower limit is naturally

$$m = 1$$

For any degree of hollowness between these two

limits, it is seen that β lies between 0 and 1 while g_1 lies between 0 and g_2.

A convenient means of expressing the "hollowing effect" is the equivalent permeability μ_e which a solid spheroid of the same outer dimensions as the hollow core (and hence same g_2) must have in order to give the same m. This is evaluated by putting

$$m = \frac{\mu_e}{1 + g_2(\mu_e - 1)} \quad .. \quad .. \quad .. \quad (16)$$

and gives

$$\mu_e = \mu \frac{\mu - g_1(\mu - 1) - \beta(1 - g_2)(\mu - 1)}{\mu - \mu g\ (\mu_1 - 1) + \beta g_2(\mu - 1)}$$
$$= \mu - \frac{\beta(\mu - 1)}{1 - (g_1 - \beta g_2)(1 - 1/\mu)}$$

If the ratio of the area of the hollow to that of the whole central cross section is

$$\gamma = \frac{b_1^2}{b_2^2}$$

the equivalent permeability is closely given by the mean of the product of area and permeability namely

$$\mu_e \approx \gamma + \mu(1 - \gamma) = \mu - (\mu - 1)\gamma \quad .. \quad (17)$$

for a core having an external axis ratio of greater than about 3 : 1.

To test the validity of this simple formula we consider the case of

$$\frac{a_2}{b_2} = 5 \quad \mu = 50 \quad \text{for which } \mu_1 = 13.3 \text{ in}$$

a solid spheroid.

Fig. 3 shows the variation of the step-up m with the area ratio γ as deduced from the exact equation, but the exact and approximate values of m agree to better than 1 per cent. everywhere.

REFERENCES

[1] F. M. Colebrook: "Magnetic Materials at Radio Frequencies." Radio Research Board Special Report No. 14, H.M.S.O., 1934.

[2] F. Ollendorff: "Magnetostatics of Mass Cores." Arch. Elektrotech., 1931, Vol. 25, pp. 436-447.

[3] M. Divilkovsky: "The Problem of a Metallic Sphere in a Uniform Alternating Magnetic Field." U.S.S.R., J. Phys., 1939, Vol. 1, pp. 471-478.

[4] M. Kornetzki and A. Weis: "The Eddy Current Losses in Mass Cores." Wiss. Veroff, a.d. Siemens-Werken, 1936, Vol. 15, pp. 95-111.

[5] W. Deutschmann: "Concerning Mass Cores." Elekt. Nachr.-Tech, 1932, Vol. 9, pp. 421-433.

[6] C. P. Edwards: "Enemy Airborne Radio Equipment." J.I.E.E., 1944, Vol. 91, Pt. III, pp. 44-66.

CORRESPONDENCE

Letters of technical interest are always welcome. In publishing such communications the Editors do not necessarily endorse any technical or general statements which they may contain.

Problem of Two Electrons

To the Editor, "Wireless Engineer."

SIR,—The apparent paradox discussed in your Editorial of January, 1946, is readily resolved by reference to the principle of special relativity according to which physical laws are invariant with respect to the Lorentz transformation.

In the case considered let the "stationary" frame of reference be S and let the frame S' move with the electrons in the x direction with velocity u. The co-ordinates of the electrons in S' are $(x', y', 0)$ and $(x', y' + r, 0)$ so that in S' the y-component of electrostatic repulsion is the only electromagnetic field component: i.e.,

$$Y' = \frac{e}{r^2}$$

If the Lorentz transformation is applied to Y' to give the fields observed in S there is the electric field

$$Y = KY' \quad \left[K = \left(1 - \frac{u^2}{c^2}\right)^{-\frac{1}{2}}\right]$$

and in addition a magnetic field in the z-direction

$$\gamma = K\frac{u}{c}Y'$$

which for u/c small corresponds to the usual formula for the magnetic field generated by crossing an electric field with uniform velocity u.

The resultant force between the electrons as observed in S is

$$eY - \frac{e\gamma u}{c} = KeY'\left(1 - \frac{u^2}{c^2}\right) = \frac{eY'}{K}$$

Since in passing from S' to S the electronic mass is transformed in the ratio K, the y component of acceleration in the ratio $1/K^2$, and the charge is invariant the equation of motion in the frame S is found to have the same form as in S'.

It is noted that the ratio of the attractive force to the repulsive force in S is u^2/c^2, and since this is always less than unity the resultant force appears to be a repulsion in any frame of reference.

This problem serves to illustrate that the resolution of an electromagnetic field into its electric and magnetic components is entirely relative to the observer and that the Lorentz transformation provides a set of relations which is useful in considering the electrodynamics of moving systems.

R. E. BURGESS.

National Physical Laboratory, Teddington.

Newton's Third Law

To the Editor, "Wireless Engineer"

SIR,—I have followed with considerable interest your Editorials relating to the forces on current elements and Newton's third law. As a consequence I have endeavoured to track down the history of the development of the Biot-Savart-Ampère formulas and have succeeded reasonably well with the exception of one link. In "A Treatise on Electricity and Magnetism" by Mascart and Joubert (translated by Atkinson), Thomas de la Rue & Co., London, 1883, to which you refer, I find on p. 441:

"V. *The action of a magnet on an element of current is applied to the element.*—This results from

the following experiment, due to M. Liouville. Part of the rectilinear current is made movable about its axis; with this object, its ends dip in two small mercury cups by which the current enters. The rectilinear element does not rotate at all, in whatever manner the magnet is presented to it."

If you or any of your readers can direct me to the account of that experiment I shall be much obliged. The experiment apparently was to test for the existence of a couple acting on a current element under the influence of a magnetic pole, the idea of which was so annoying to Ampère. However, whether this experiment actually entered into Ampère's argument or was made sometime after his work was done is a point on which I am not satisfied. I get the impression from the presentation of arguments by Mascart and Joubert that they outline the way Ampère might have reasoned with the hindsight available in 1883. I cannot directly trace all of the Mascart-Joubert logic in Ampère's papers. The reference they make to Liouville does not clearly say whether he performed the experiment or merely suggested it. Joseph Liouville (who presumably is meant) appears never to have done any experimental work, only pure mathematical analysis, according to his listed papers, and his one paper (1829) dealing with current elements does not suggest the experiment. The few others of the name Liouville who have works listed prior to 1883 do not deal with the subject, as far as I can ascertain.

R. H. Frazier.

Associate Professor, Massachusetts Institute of Technology, U.S.A.

Preferred Numbers and Filter Design

To the Editor, "Wireless Engineer."

Messieurs,—Dans le numéro 265 de votre Revue (Octobre 1945), j'ai relevé un article de M. H. Jefferson, qui a retenu toute mon attention.

Cet article en effet fait ressortir les avantages importants que peuvent procurer l'application des séries géométriques décimales de nombres pour caractériser les éléments normalisés. Toutefois, l'auteur suggère des séries comportant respectivement, 6, 12 et 24 termes entre 2 puissances successives de 10, c'est-à-dire de raisons $\sqrt[6]{10}$, $\sqrt[12]{10}$ — $\sqrt[24]{10}$:

10 — 15 — 22 — 33 —
10 — 12 — 15 — 18 — 22 — 27 — 33 — 39
10–11–12–13–15–16–18–20–22–24–27–30–33–36–39–

47 — 68 — 100
— 47 — 56 — 68 — 82 — 100
43–47–51–56–62–68–75–82–91–100

Ayant eu l'honneur de présider les réunions du Comité International de l'I.S.A. qui avait été chargé, en 1932, d'étudier la normalisation des nombres abstraits, je me permets de vous signaler les résultats des études qui avaient été entreprises avec le concours des Associations de Normalisation de 19 nations[1]:

Prenant comme base les remarquables travaux effectués en 1877 par le Français RENARD, alors capitaine du Génie, l'accord international de Stockholm (1934) consacre les séries géométriques décimales dites "Séries RENARD" comportant respectivement 5–10–20 & 40 termes entre deux puissances successives de 10, c'est-à-dire de raisons:

$\sqrt[5]{10}$ $\sqrt[10]{10}$ $\sqrt[20]{10}$ $\sqrt[40]{10}$

Le document ci-joint[2] donne les valeurs correspondantes, ainsi que les valeurs calculées des nombres des séries géométriques.

Les nombres normaux (Preferred Numbers) ainsi définis ont d'ailleurs été extraits:

tantôt de la norme française de Renard utilisée depuis des dizaines d'années, en construction aéronautique principalement;

tantôt de la norme allemande établie en 1923 par la DIN*.

En fait, ils ont été choisis pour serrer de plus prés les valeurs exactes (les écarts maximum sont — 1,01% et + 1,26%); ils possèdent évidemment les mêmes avantages que ceux que Monsieur Jefferson signale dans son article, mais ils permettent en outre des facilités de calcul qui sont exposées en détail en page 6 du document joint.

J'ajoute que dès 1936, l'A.S.A. (AMERICAN STANDARDS ASSOCIATION) a publié les mêmes séries géométriques décimales que celles de l'I.S.A., complètées par des séries analogues pour le système de numération correspondant au pouce, à ses multiples et à sous-multiples.

J'estime donc qu'il y aurait un intérêt de premier order à ce que tous les travaux de normalisation faisant appel aux séries géométriques décimales s'appuient sur les séries RENARD internationales et je serais heureux de recevoir à ce sujet votre avis ainsi que celui de votre collaborateur.

P. Blanchet.

Compagnie Francaise Thomson-Houston, Paris.

To the Editor, "The Wireless Engineer."

Sir,—I am grateful to M. Blanchet for drawing my attention to the work of Colonel Renard. The Renard series clearly obeys the basic definition given on page 484 of my paper and is particularly convenient in that the logarithms of the "Renard numbers" are all multiples of 0.025. The artifice of using logarithms to a base of 1.47 is therefore avoided.

The series used in my paper is, of course, that adopted during the war for the rationalization of British components. The object of my paper was to arrive at a solution in this series: the method is, as M. Blanchet points out, equally applicable to any scheme of preferred numbers.

H. Jefferson.

London, S.W.1.

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[1] Les Associations nationales de normalisation allemande, autrichienne, belge, danoise, finlandaise, française, hollandaise, norvégienne, polonaise, suédoise, suisse, tchécoslovaque, U.R.S.S., avaient adhéré à l'accord international de 1934. L'Association japonaise s'était déclarée favourable par lettre. Les Associations espagnole, hongroise et italienne, sans faire obstacle au projet, avaient simplement réservé leur accord définitif.

[2] Fascicule XI-00 d'Août 1942, de l'Association Française de Normalisation (AFNOR).

* DIN (Deutsche Industrie Normen.)—[Ed.]

WIRELESS PATENTS

A Summary of Recently Accepted Specifications

The following abstracts are prepared, with the permission of the Controller of H.M. Stationery Office, from Specifications obtainable at the Patent Office, 25, Southampton Buildings, London, W.C.2, price 1/- each.

TRANSMITTING CIRCUITS AND APPARATUS

573 957.—Oscillator-unit, comprising a grid-controlled valve coupled to a wave-guide, for generating centimetre waves.

The General Electric Co. Ltd. and E. G. James. Application date 11th February, 1943.

573 981.—Cathode-circuit modification of a short-wave oscillator comprising two push-pull valves feeding a concentric-line resonator.

The General Electric Co. Ltd. and M. R. Gavin. Application date 5th November, 1940.

574 051.—Reflectionless plug-and-socket connection for concentric lines carrying centimetre waves.

The General Electric Co. Ltd. and R. J. Clayton. Application date 7th January, 1942.

574 127.—Plug-and-socket joint, forming a half-wave section, for connecting a pair of concentric cables carrying centimetre waves.

The General Electric Co. Ltd. and R. J. Clayton. Application date 7th January, 1942.

SIGNALLING SYSTEMS OF DISTINCTIVE TYPE

573 900.—Signalling by a beam of light or infra-red rays through an arrangement of modulating shutters, which are made to overlap cyclically under the control of Morse or speech.

H. S. Molyneux-Ffennell, E. T. J. Tapp, F. G Charlton, and Vacuum-Science Products, Ltd. Application date 25th August, 1941.

574 133.—Delay-network for generating pulsed signals which are predetermined time-derivatives of an input or triggering pulse, and are of given shape and amplitude.

D. Blumlein (legal representative of A. D. Blumlein). Application dates 17th June and 21st July, 1942.

CONSTRUCTION OF ELECTRONIC-DISCHARGE DEVICES

573 901.—Electrode-system for focussing to a straight line the electron-stream of a cathode-ray or like discharge-tube.

O. E. H. Klemperer (commonly known as O. Klemperer). Application date 24th March, 1942.

573 999.—Making glass-to-glass seals, in the manufacture of thermionic valves, by radio-frequency heating.

Standard Telephones and Cables Ltd., P. K. Chatterjea, and S. J. Powers. Application date 22nd October, 1943.

574 009.—Amplifier or oscillator valve in which the input circuit forms part of a hollow resonator which is arranged symmetrically about the grid of the valve.

C. S. Bull. Application date 21st June, 1941.

574 056.—Electrode lens-system for focussing an electron-stream about a straight line.

O. E. H. Klemperer (commonly known as O. Klemperer). Application date 8th April, 1942.

574 109.—Multiple anode for generating a desired spectrum of X-rays, particularly for use in diffraction analysis.

The British Thomson-Houston Co., Ltd. Convention date (U.S.A.) 11th January, 1943.

574 237.—Process of "gettering" electron-multipliers on which metallic antimony or rubidium is used to remove excess alkali metal from the secondary-emission electrodes.

Cinema-Television Ltd., and A. Sommer. Application date 19th January, 1944.

574 248.—Metal-to-glass seal for an electron discharge tube.

Standard Telephones and Cables Ltd., and S. J. Powers. Application date 21st January, 1944.

574 383.—Construction of cathode core for an electron-discharge tube, designed to prevent damage to the sensitized coating in the process of assembly.

The M-O Valve Co., Ltd., G. F. Klepp, and J. A. Smyth. Application date 2nd April, 1943.

SUBSIDIARY APPARATUS AND MATERIALS

573 660.—Radio-frequency screening-device made of rubber mixed with a high carbon-content, to reduce its electrical resistance.

J. J. Davis. Application date 29th November, 1940.

573 679.—Continuously-adjustable choke-coil, incorporating a stationary magnetic circuit and a movable winding.

Akt. Brown, Boveri et Cie. Convention date (Switzerland) 23rd December, 1942.

573 717.—Transformer with two saturable paths, and a relay, for automatically generating a sub-harmonic of the triggering frequency.

Automatic Electric Laboratories Inc. Convention date (U.S.A.) 6th May, 1943.

573 793.—Amplifier, for use with a photo-electric cell or otherwise, comprising a saturated reactance coil and a crystal rectifier providing negative feedback for voltage-control.

Standard Telephones and Cables Ltd. (communicated by International Standard Electric Corporation). Application date 6th November 1943.

573 936.—Electrical processing of a dry-contact rectifier to build-up its reverse resistance.

Standard Telephones and Cables Ltd., B. B. Grace, L. J. Ellison, H. S. Leman, C. W. Leng, and A. S. Bridge. Application date 15th February, 1943.

573 959.—Circuit arrangement for giving a continuous indication of the reverse resistance of a dry-contact rectifier during the final processing of the rectifier.

The British Thomson-Houston Co., Ltd., and J. Dyson. Application date 29th March, 1943.

574 345.—Logarithmic ready-reckoner for electrical calculations, comprising three coaxial rotary discs and a cursor.

K. M. Bancroft and Ionic Laboratories Ltd. Application date 25th January, 1944.